Voting on Redistribution under Quasi-Maximin Altruism

Sabine Flamand

Supervisor: Caterina Calsamiglia

International Doctorate in Economic Analysis
Universitat Autònoma de Barcelona

June 3, 2009

Abstract

We introduce fairness as quasi-maximin political preferences in a standard model of voting on redistribution. We show that the presence of fair voters does not necessarily imply a higher level of redistribution than in the case of purely self-interested voters. We study the link between inequality and redistribution, and show that allowing for fairness, the model yields different predictions than in the case of selfishness. We examine the implications of assuming a mixture of fair and selfish voters in a three-voter economy and identify the cases for which the identity of the median voter might be altered. Finally, we compare our results with the ones obtained assuming instead self-centered inequality aversion à la Fehr and Schmidt (1999).

1. Introduction

Recent experimental evidence indicates that individual concerns for fairness and altruism can explain a range of economic phenomena which are not in accordance with the traditional selfishness assumption. Other-regarding preferences may take several forms: altruism, inequity aversion and reciprocity (see Fehr and Schmidt (2006) for a recent survey). In this paper, we examine the implications of introducing fair voters who have preferences for fairness as in Charness and Rabin (2002) in a standard model of majority voting on a redistributive parameter. More specifically, we endow individuals that have different skill levels with quasi-maximin political preferences, i.e. preferences that include both a utilitarian and a rawlsian motive when voting on the redistributive policy. This contrasts with the existing theoretical literature on fairness and voting on redistribution, which has mainly modeled fairness as self-centered inequity aversion.

By introducing this specific type of altruism into a redistributive context, we would like to answer several questions: first, what is the political equilibrium on redistribution, if one exists, when voters are endowed with this specific form of altruism? We show that, depending on the weights associated to the social welfare motives in the voter's utility function, the Condorcet winner tax rate of an economy with fair voters might be smaller than the one of an economy in which voters are selfish. Second, the benchmark model with selfish voters predicts that higher inequality leads to more redistribution in equilibrium. Empirically, this relation remains controversial. Does the model with fairness yield the same theoretical predictions regarding this issue? Third, experimental evidence points out that people are heterogeneous with respect to their preferences for fairness. In particular, it seems that roughly 50 percent of the population behaves completely selfishly. Hence, a question that naturally arises is whether mixing fair and selfish voters have interesting implications for the resulting equilibrium. Using a simple 3-voter economy, we identify several cases for which the identity of the median voter might be altered relatively to the case of an economy composed exclusively of either fair or selfish voters. In all the cases, the redistributive outcome ends up being controlled by a fair voter. Finally, in the last section, we compare our results with the ones obtained by Dhami and al-Nowaihi (2008a and b) who did exactly the same exercise using instead self centered inequity aversion *à la* Fehr and Schmidt (1999).

2. Social Preferences and Voting on Redistribution

The standard approach on redistribution through the voting process comes from the models of Romer (1975), Roberts (1977) and Meltzer and Richards (1981). The commonly used name for this class of models is the RRMR model. It is a general equilibrium model which assumes purely selfish individuals that differ with respect to their productivity, which determines their income. It predicts that the extent of redistribution is determined by the median-income voter, with higher-income individuals preferring less taxation than lower income individuals. More specifically, the median voter, being selfish, will vote for a strictly positive tax rate only to the extent that he is poorer than average. As a consequence, the model also predicts that higher inequality, as measured by the median-to-mean income ratio, goes together with higher redistribution in equilibrium.

A few theoretical papers have recently introduced social preferences into models of voting on redistribution. Galasso (2003) introduces inequity averse fair agents in the Meltzer and Richard (1981) framework. The author models fairness as self-centered inequality aversion that is one-sided, using the poorest agent in the economy as the reference. Hence, in this set up, a voter dislikes being richer than the poorest voter, but he does not mind being poorer than other voters (i.e. there is no envy). Dhami and al-Nowaihi (2008a and b) also use the traditional RRMR framework in order to allow for fairness. The utility function they use is the one proposed by Fehr and Schmidt (1999), reflecting two-sided self-centered inequality aversion. This means that a voter dislikes both being poorer (disadvantageous inequality) and richer (advantageous inequality) than other voters, as he suffers disutility from any income difference between himself and the other individuals (i.e. there is both envy and altruism). In both papers, the existence of fair voters increases the equilibrium level of redistribution relatively to the benchmark model assuming purely selfish voters. Using another approach, Tyran and Sausgruber (2006) also use inequity aversion as in Fehr and Schmidt (1999) in order to study voting on redistribution. They test their predictions and find that the model with fair voters predicts voting outcomes far better than the standard model of voting assuming rationality and strict self-interest.

Those papers share the common feature of modeling fairness as self-centered inequality aversion, whether one- or two-sided. However, individual concerns for fairness in a context of redistribution might well not be of the difference aversion type, but rather be sensitive to some other, general, notion of social welfare. In this paper, we will use social preferences as suggested by Charness and Rabin (2002). They combine altruistic preferences with a specific form of inequity aversion that they call *quasi-maximin* preferences. An individual's overall utility function is given by a convex combination of his own monetary payoff and a social welfare function that is itself a convex combination of rawlsian and utilitarian altruism (this last part, apart from reflecting surplus maximization, also reflects altruism based on the idea that each individual's payoff receives the same weight):

$$U_i\left(x_1, ..., x_n\right) = \left(1-\lambda\right) x_i + \lambda \left[\delta \min\left\{x_1, ..., x_n\right\} + \left(1-\delta\right) \frac{1}{n} \sum_{i=1}^{n} x_i\right]$$

The case $\lambda = 1$ corresponds to the purely disinterested case, whereas the case $\lambda = 0$ corresponds to the purely self-interested case. Note that, in contrast with the original utility function proposed by Charness and Rabin (2002), in which utilitarianism is modeled as the sum of individual payoffs, we assume instead that individuals care about the mean of individual payoffs, so as to have an utility function that is normalized with respect to the number of individuals in the economy.

Charness and Rabin (2002) provide strong experimental justification of social welfare models over self-centered models. Their data show that social welfare preferences explain behavior better than difference aversion. In other words, individuals are not indifferent to the distribution of payoffs among *other* people. More specifically, the authors find strong degree of respect for social efficiency, tempered by concern for those less well off. Similarly, using simple one-shot distribution experiments, Engelmann and Strobel (2004) show that influence of both efficiency and maximin preferences is stronger than that of inequality aversion.

With quasi-maximin preferences, people like to increase the social surplus (utilitarian motive) but at the same time they care especially about individuals with low payoffs (rawlsian motive). Charness and Rabin (2002) also find strong evidence of reciprocal behavior, but such a behavior is obviously more likely to occur in strategic settings, where players can directly affect each other's payoffs (in the case of bilateral interactions for example). Hence, it might be less relevant in a voting framework. As for the relevance of the maximin criterion, Fehr and Schmidt (2006, p. 52) point out that "[...] the maximin motive may be more or even highly relevant in the context of charitable giving or in the context of referenda or elections with a large number of people, where strategic voting is unlikely to occur". Engelmann and Strobel (2004) also emphasize the relevance of the maximin motive in multi-person dictator games.

3. The Model

We introduce fairness in the standard RRMR framework, in which the proceeds from a linear tax are used to finance equal per capita transfers to all voters.

We assume that there are $n = 2m - 1 \geq 3$ voters, where m is the median voter. The voters are differentiated by their ability level, which is also their wage rate, denoted by ω_i. Thus, the skill of voter i is given by ω_i where $0 < \omega_i < \omega_j < 1$ and $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_n)$. We make the assumption that the ability of the median voter, ω_m, is smaller than the average ability $\overline{\omega}$.

Each individual is endowed with a fixed time endowment of one unit and supplies l_i units of labor and so enjoys $L_i = 1 - l_i$ units of leisure, where $0 \leq l_i \leq 1$. Labor markets are competitive and each firm has access to a linear production technology such that production equals $\omega_i l_i$. Hence, the wage rate offered to each worker-voter coincides with the marginal product, i.e. the skill level ω_i. Therefore, the before-tax income of a voter is given by $y_i = \omega_i l_i$. The assumption made on abilities, that is, the fact that $\omega_m < \overline{\omega}$, will obviously also translates into incomes. This assumption is empirically plausible as typical income distributions are skewed to the left. Hence, the following strict inequality holds:

$$y_m < \frac{1}{n}\sum_{i=1}^{n} y_i = \overline{y}$$

The budget constraint of voter i is given by

$$0 \leq c_i \leq (1-t)\, y_i + b$$

$$\Leftrightarrow 0 \leq c_i \leq (1-t)\, \omega_i l_i + b$$

where $t \in [0,1]$ is the tax rate and b is the uniform transfer given to each voter that equals the average tax proceeds ($b = t\overline{y}$).

The sequence of moves is the following: we consider a two-stage game. In the first stage, voters choose a tax rate, t, anticipating the outcome of the second stage. Consumer i exhibits fairness by voting for the tax rate that would maximize social welfare as seen from his own perspective (see below). In the second stage, consumer i chooses own labor supply l_i so as to selfishly maximize own utility. This determines the vector of labor supplies and indirect utilities.

3.1. Individual choice of labour supply (second-stage game)

Taking the redistributive policy of the government as given (i.e. t and b), labor supply is determined on the basis of private preferences. Voter i has a utility function $U(c_i, 1-l_i)$, over own consumption, c_i, and own leisure, $1-l_i$. All voters have the same utility function. Hence, voters differ only in that they are endowed with different skill levels, ω_i. Following the literature, utility is assumed to be a quasi-linear utility function of the form[1] $U(c_i, 1-l_i) = c_i + u(1-l_i)$.

The optimization problem of individual i is given by

$$\underset{l_i}{Max}\ U(c_i, 1-l_i) \text{ such that } 0 \leq c_i \leq (1-t)\, \omega_i l_i + b$$

[1] Note that assuming such linearity in consumption has an important implication regarding the utilitarian concerns of voters. When individuals' utility function is linear in c, maximizing the sum/average of utilities (i.e. utilitarianism) will be independent of any distributional concerns. In contrast, if one assumes an individual utility function that is strictly concave in c, maximizing the size of the pie requires perfect equality of consumption among individuals. Besides the fact that such (quasi)- linearity is convenient for tractability of analysis, it reflects the assumption according to which utilitarianism might only entail efficiency concerns, the distributional concerns here being reflected by the rawlsian motive that is also present in the voter's utility function.

$$\Leftrightarrow \underset{l_i}{Max}\ U\left(\left(1-t\right)\omega_i l_i + b, 1 - l_i\right)$$

The economic optimization problem yields the usual result

$$\left(1-t\right)\omega_i = u'\left(1 - l_i^*\right)$$

which implicitly defines l_i^* as a function of ω_i and t, and thus $y_i^* = \omega_i l_i^*$.

Following the literature, we assume that the quasi-linear utility function takes the following quadratic form in leisure:

$$U\left(c_i, 1 - l_i\right) = c - \frac{1}{2}l^2$$

The utility function $U\left(c_i, 1 - l_i\right)$ is twice differentiable, strictly concave in leisure, and the marginal utility of both consumption and leisure are positive:
i) $\frac{\partial U(c_i,1-l_i)}{\partial c} = 1 > 0$
ii) $\frac{\partial U(c_i,1-l_i)}{\partial l} = -l < 0$
iii) $\frac{\partial^2 U(c_i(l_i),1-l_i)}{\partial l_i^2} = -1 < 0$

The first order condition with respect to l_i yields

$$l_i^* = \left(1-t\right)\omega_i$$

$$y_i^* = \omega_i l_i^* = \left(1-t\right)\omega_i^2$$

Hence, private preference satisfaction is measured by the indirect utility function

$$\upsilon_i = \left(1-t\right)\omega_i l_i^* + b + u\left(l_i^*\right)$$

$$\Leftrightarrow \upsilon_i = \upsilon\left(t, b, \omega_i\right) = \frac{1}{2}\left(1-t\right)^2\omega_i^2 + b$$

The indirect utility function $\upsilon\left(t, b, \omega_i\right)$ satisfies the following properties:
i) $\frac{\partial \upsilon(t,b,\omega_i)}{\partial b} = 1 > 0$
ii) $\frac{\partial \upsilon(t,b,\omega_i)}{\partial t} = -\left(1-t\right)\omega_i^2 < 0$
iii) $\frac{\partial \upsilon(t,b,\omega_i)}{\partial \omega_i} = \left(1-t\right)^2\omega_i > 0$

Substituting for $b = \frac{t}{n}\sum\limits_{i=1}^{n} y_i = \frac{t}{n}\sum\limits_{i=1}^{n}\omega_i l_i^* = \frac{t}{n}\sum\limits_{i=1}^{n}\omega_i^2\left(1-t\right)$, we finally have that

$$\upsilon_i = \upsilon\left(t, \omega_i\right) = \frac{1}{2}\left(1-t\right)^2\omega_i^2 + \frac{t\left(1-t\right)}{n}\sum_{i=1}^{n}\omega_i^2$$

3.2 Voting for a tax rate under quasi-maximin altruism (first-stage game)

All fair agents take their voting decisions by maximizing their quasi-maximin indirect utility function, which is purely in terms of the tax rate. The indirect utility function of a fair voter i is given by

$$V_i(\upsilon_1, ..., \upsilon_n) = (1-\lambda)\upsilon_i + \lambda\left[\delta \min\{\upsilon_1, ..., \upsilon_n\} + (1-\delta)\frac{1}{n}(\upsilon_1 + ... + \upsilon_n)\right]$$

As already mentioned, an individual's overall utility function is given by a convex combination of his own indirect utility function and a social welfare function which is itself a convex combination of rawlsian and utilitarian altruism.

Substituting for the expression of υ_i found above in $V_i(\upsilon_1, ..., \upsilon_n)$ gives $V_i(t, \omega_1, ..., \omega_n)$. Then, taking derivative with respect to t and setting this quantity equal to zero yields the preferred tax rate[2] of voter i:

$$t(\omega_i) = \frac{(1-\lambda)\left(\frac{1}{n}\sum_{i=1}^{n}\omega_i^2 - \omega_i^2\right) + \lambda\delta\left(\frac{1}{n}\sum_{i=1}^{n}\omega_i^2 - \omega_1^2\right)}{(1-\lambda)\left(\frac{2}{n}\sum_{i=1}^{n}\omega_i^2 - \omega_i^2\right) + \lambda\delta\left(\frac{1}{n}\sum_{i=1}^{n}\omega_i^2 - \omega_1^2\right) + \lambda\frac{1}{n}\sum_{i=1}^{n}\omega_i^2} \in (0,1)$$

Notice that we can transform this equation in order to express the tax rate as a function of individual's gross incomes (recall that $y_i^* = \omega_i l_i^* = (1-t)\omega_i^2$):

$$t(y_i) = \frac{(1-\lambda)(\overline{y} - y_i) + \lambda\delta(\overline{y} - y_1)}{(1-\lambda)(2\overline{y} - y_i) + \lambda\delta(\overline{y} - y_1) + \lambda\overline{y}}$$

3.3. The equilibrium and its properties

In pairwise votes over proposals in one dimensional issue space, an equilibrium can be shown to exist if preferences satisfy single-peakedness. As the second derivative of $V_i(t, \omega_i)$ with respect to t is strictly negative for all i, the indirect utility function satisfies single-peakedness on the t dimension for all i. Therefore, the median voter's preferred tax rate is the Condorcet winner tax rate.

Proposition 1: Political equilibrium

(a) The Condorcet winner tax rate in the fair economy is the tax rate chosen by the median-income voter and is given by

$$t_m^F = \frac{(1-\lambda)(\overline{y} - y_m) + \lambda\delta(\overline{y} - y_1)}{(1-\lambda)(2\overline{y} - y_m) + \lambda\delta(\overline{y} - y_1) + \lambda\overline{y}}$$

[2] See appendix.

(b) The Condorcet winner tax rate in the selfish economy is the tax rate chosen by the median-income voter and is given by

$$t_m^S = \frac{\overline{y} - y_m}{2\overline{y} - y_m}$$

Proof: in appendix.

The expression for t_m^S illustrates the celebrated result according to which the extent of redistribution varies directly with the ratio of mean-to-median income. In contrast, it is clear from the expression of t_m^F that the extent of redistribution in the fair economy depends on other parameters, among which the poorest individual's income, or both the fairness parameters λ and δ. Proposition 2 below gives the change in the Condorcet winner tax rates for the fair and selfish economies following marginal increases in various parameters of the model.

Proposition 2: **Comparative statics**

(a) The fair Condorcet winner tax rate is increasing in δ *(*$\frac{\partial t_m^F}{\partial \delta} > 0$*)*

(b) The fair Condorcet winner tax rate is increasing in λ *if and only if* δ *is "high enough" (*$\frac{\partial t_m^F}{\partial \lambda} > 0$ *if and only if* $\delta > \frac{\overline{y} - y_m}{\overline{y} - y_1}$*)*

Corollary: the fair Condorcet winner tax rate is higher than the selfish Condorcet winner tax rate if and only if $\delta > \frac{\overline{y} - y_m}{\overline{y} - y_1}$

(c) The selfish Condorcet winner tax rate is decreasing in own income ($\frac{\partial t_m^S}{\partial y_m} < 0$*)*

(d) The fair Condorcet winner tax rate is decreasing in own income if and only if $\frac{(1-\lambda)}{\lambda\delta} > \frac{y_1}{n\overline{y} - y_m}$ *(*$\frac{\partial t_m^F}{\partial y_m} < 0$ *if and only if* $\frac{(1-\lambda)}{\lambda\delta} > \frac{y_1}{n\overline{y} - y_m}$*)*

(e) The fair and selfish Condorcet winner tax rates are increasing in mean income ($\frac{\partial t_m^F}{\partial \overline{y}} > 0$ *and* $\frac{\partial t_m^S}{\partial \overline{y}} > 0$*)*

(f) The selfish Condorcet winner tax rate is increasing in any voter's income ($\frac{\partial t_m^S}{\partial y_j}|_{j \neq m} > 0$*)*

(g) The fair Condorcet winner tax rate is increasing in any voter's income except the poorest one ($\frac{\partial t_m^F}{\partial y_j}|_{j \neq m,1} > 0$*)*

(h) The fair Condorcet winner tax rate is decreasing in the poorest voter's income if and only if $\frac{\lambda\delta}{(1-\lambda)} > \frac{y_m}{n\overline{y} - y_1}$ *(*$\frac{\partial t_m^F}{\partial y_1} < 0$ *if and only if* $\frac{\lambda\delta}{(1-\lambda)} > \frac{y_m}{n\overline{y} - y_1}$*)*

Proof: in appendix.

In an economy in which fair voters have quasi-maximin preferences, the Condorcet winner tax rate is increasing in the parameter δ, the weight associated to the maximin criterion (part (a)). Indeed, an increase in δ increases the utility of redistributing resources towards the poorest individual, which requires an increase in the redistributive parameter t_m^F. From part (b), an increase in the weight associated to the social welfare criterion λ induces an increase in the tax rate t_m^F if and only if $\delta > \frac{\overline{y}-y_m}{\overline{y}-y_1}$. An increase in λ has two distinct effects. First, it increases the utility of redistributing to the poorest through the weight $\lambda\delta$, which requires a higher tax rate. Second, it increases the utility of maximizing the surplus through the weight $\lambda(1-\delta)$. This second effect has two components. While an increase in the redistribution towards the poor individuals increases their utility and hence the surplus, the maximization of this same surplus requires that the distortions associated to taxation are minimized, and thus requires a decrease in the tax rate. Furthermore, there is an altruistic cost of taxing the rich. Therefore, a higher λ induces a higher equilibrium tax rate if and only if δ is high enough, the threshold value being a function of the difference between the median and average incomes and between the median and lowest incomes in the economy: the bigger the difference between the median and mean incomes, and the lower the difference between the median and lowest incomes, the bigger the required δ in order to induce an increase in redistribution following an increase in λ. Suppose that y_1 increases. As a result, $(\overline{y}-y_m)$ increases and $(\overline{y}-y_1)$ decreases. As the increase in y_1 imples a higher average income, the selfish median voter would like to redistribute more. However, the fair median voter would like to redistribute less (provided that the corresponding condition is satisfied (Proposition 2h)) Those two facts make it less likely that $t_m^F > t_m^S$. Therefore, a higher δ is required in order to satisfy this inequality. Finally, notice that, for the poorest individual, part (b) reads $\frac{\partial t_1}{\partial \lambda} > 0$ if and only if $\delta > 1$. Hence, following the same reasoning, it turns out that the selfish poorest individual will vote for a higher tax rate that the poorest fair individual.

From parts (c) and (d), selfish and fair median voters would like to redistribute less when they get richer. For the fair median voter, this is true if and only if the fairness parameters are such that $\frac{(1-\lambda)}{\lambda\delta} > \frac{y_1}{n\overline{y}-y_m}$. In other words, given the income distribution, if λ and δ are low enough, so that the condition is fulfilled, the fair median voter will prefer a lower tax rate as his own income increases. From part (e), an increase in the average income translates into an increase in the tax rate preferred by both the selfish and fair median voters. As can be seen from the expression for t_m^S, the selfish median voter would like to have positive redistribution only to the extent that he is poorer than the average voter. Similarly, from part (f), an increase in any voter's income will increase the selfish median voter's preferred tax rate as this will translate into an increase in the average income. As a consequence, in an economy with selfish voters, an increase in poverty (a decrease in y_j for $j < m$) will decrease the equilibrium level of redistribution[3]. This is also true for an economy with fair voters (part (g)): an increase in any voter's income - except the poorest - will increase the tax rate chosen by the median voter. However, this is not any longer true for the case of an increase in the poorest individual's income (part (h)). Indeed, when y_1 increases, the Condorcet winner tax rate decreases

[3] This issue will be addressed in the next section on the link between inequality, fairness and redistribution.

if and only if $\frac{\lambda\delta}{(1-\lambda)} > \frac{y_m}{n\overline{y}-y_1}$. Hence, in other words, if the fairness parameters are high enough given the income distribution, an increase in poverty increases redistribution in this case and vice versa. Finally, when y_j increases for $j > m$, both the selfish and fair median voters would like to increase the redistributive tax rate. In other words, when the rich get richer, both kinds of voters respond with increased redistribution. Notice that in practice, n might be big enough so that both parameter restrictions in (d) and (h) are satisfied, so that $\frac{\partial t_m^F}{\partial y_m} < 0$ and $\frac{\partial t_m^F}{\partial y_1} < 0$.

In the next section, we will focus with more details on the link between inequality and redistribution in the context of quasi-maximin altruism. In order to do so, we examine the implications of assuming different ways of increasing the inequality of income distribution for the equilibrium tax rate.

4. Fairness, inequality and redistribution

In this part, we will explore how the introduction of quasi-maximin altruism in the model affects the link between income inequality and redistribution. The RRMR model predicts that higher inequality (lower median-to-mean income ratio) implies higher redistribution. Empirically, this remains controversial. The most cited counter-example comes from the comparison between Europe and the United States, where Europe typically has lower pre-tax inequality together with more redistribution. Borck (2007, p. 96), in his survey on voting, inequality and redistribution, discusses this issue and concludes that "the RRMR hypothesis of a link between inequality and the size of the government has met with mixed empirical evidence". More generally, the literature has highlighted several points regarding this issue, among which the following:

(a) Social spending might increase with growing inequality in some but not all categories (political power of some interest groups, inter- versus intra-generational tranfers of income, etc.) (Borck (2007)).

(b) The POUM hypothesis: the "Prospect of Upward Mobility" implies that low levels of redistribution are consistent with high pre-tax income inequalities (Benabou and Ok (2001)).

(c) Beliefs on the determinants (luck versus effort) of pre-tax income inequalities might be correlated with levels of redistribution. Hence the differences between Europe and the United States (Alesina et al. (2001)).

(d) Data and research design: several factors being different across countries might potentially affect the empirical link between inequality and the extent of redistribution, many of which cannot be controlled for (Acemoglu and Robinson (2005)). Moreover, empirical studies might not have used the required data on income distribution so far (factor versus disposable income) (Milanovic (2000)).

Both the theoretical and empirical literature on this issue are extensive, and it is beyond the scope of this paper to fully review and discuss it. Rather, we would like to know what are the theoretical predictions on the link between inequality and redistribution in the context of the present model. In other words, we would like to determine whether introducing quasi-maximin altruism into the standard RRMR framework yields different predictions regarding this link.

4.1. Inequality as a variation in average income

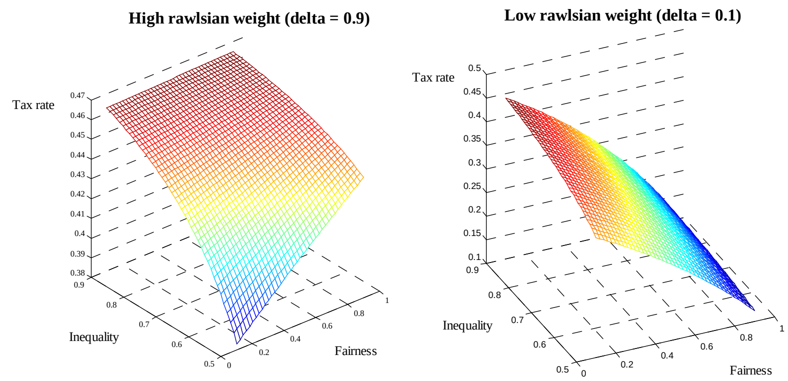
The first question we address is the following: what is the effect of increased poverty on redistribution? In the standard RRMR model with selfish voters, an increase in poverty reduces the equilibrium level of redistribution. This is so because the increased poverty reduces the mean income in the economy, and hence, the median remaining unchanged, the median-to-mean income ratio increases correspondingly. When we introduce fairness as quasi-maximin preferences, we saw in our comparative statics results that the Condorcet winner tax rate is decreasing following a decrease in any below-median income except the poorest one. Hence, the introduction of fair voters in this case does not change the result according to which an increase in poverty reduces the equilibrium level of redistribution. In the same way, when the rich gets richer, the mean income in the economy increases, which increases the equilibrium level of redistribution in both the standard model and the model with fair voters. Insofar as periods of increased poverty are associated with slower economic activity, the prediction is that social spending is pro-cyclical in both economies (selfish and fair). However, we saw that this is not the case as far as the poorest individual is concerned in the fair economy[4]. In practice, this poorest individual might represent a specific category of individuals in the society, whose social benefits might increase in periods of slower economic activity (i.e. be counter-cyclical).

In order to illustrate the link between fairness, inequality and redistribution in our model, we plot, in figure 1, the tax rate chosen by the median-income individual against both fairness and inequality. In this exercise, an increase in inequality is generated by making the rich even richer, whereas an increase in fairness is generated by λ getting higher[5]. As can be seen from the graph, and as we already mentioned, for a given level of inequality, higher fairness implies higher redistribution when δ is high, and higher fairness implies lower redistribution when δ is low (Proposition 2b). Furthermore, redistribution increases with inequality (as defined by the rich getting richer) in both figures. When δ is high, it appears that low-inequality and high-fairness countries have a similar (even though a bit smaller) level of redistribution as high-inequality and low-fairness countries. In this case, controlling for fairness seems important if one attempts to find an empirical relation between inequality and the extent

[4] We assume that n is big enough so that $\frac{\partial t_m^F}{\partial y_1} < 0$.

[5] More specifically, the graph represents a 3-voter economy (low, median and high income) such that $y_m < \overline{y}$. We choose $\{\omega_1, \omega_2\} = \{0.1, 0.2\}$ and let ω_3 vary between 0.5 and 0.9. The rawlsian weight is given the value 0.9 and 0.1 in the first and second figures respectively. Finally, we let the fairness parameter λ vary between 0.1 and 0.9.

Figure 1: The relation between fairness, inequality and redistribution



of redistribution in a cross-section of countries. If one controls for fairness, the model predicts that higher inequality should lead to higher redistribution in equilibrium, whatever the value of δ. The magnitude of this effect will be lower or higher depending on the parameter values.

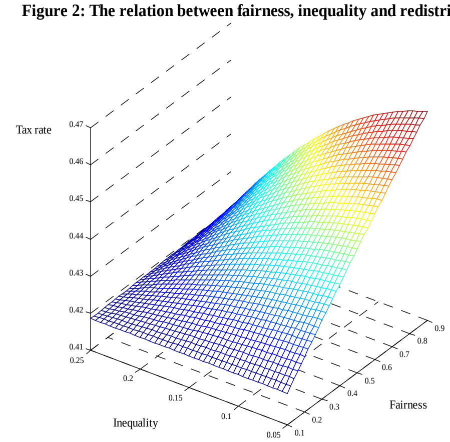
On figure 2, we repeat the same exercise, generating inequality by making the poor even poorer[6]. As can be seen from the graph, when λ is high, an increase in inequality translates into a sharp increase in the tax rate chosen by the median voter. As long as λ gets smaller, this positive link remains but with a progressively decreasing slope (remember that poverty increases redistribution if and only if the fairness parameters are high (Proposition 2h)). Note also that controlling for inequality, the tax rate increases with fairness as we chose a high value of δ.

4.2. Inequality as a mean-preserving spread

More generally, what is the relation between inequality and redistribution when one before-tax distribution is more unequal than another? In both the cases of the poor getting poorer and the rich getting richer, the consequences of such changes are driven by the corresponding variation of the average income in the economy. Hence, the following question arises: what is the relationship between inequality and redistribution when considering an increase in inequality that leaves the mean unaffected (i.e. a mean-preserving spread). To answer this question and check the various cases, the definition of a mean-preserving spread

[6] More specifically, the graph represents a 3-voter economy (low, median and high income) such that $y_m < \bar{y}$. We choose $\{\omega_2, \omega_3\} = \{0.3, 0.9\}$ and let ω_1 vary between 0.05 and 0.25. The rawlsian weight is given the value 0.9. Finally, we let the fairness parameter λ vary between 0.1 and 0.9.

Figure 2: The relation between fairness, inequality and redistribution



has to be refined somehow. We use the concept of median-dominance suggested by Dhami and al-Nowaihi (2008b) in the context of inequity aversion, and we adapt it to the case of quasi-maximin altruism.

Consider two distinct income distributions $\mathbf{x}$ and $\mathbf{y}$ with the same mean μ. In both distributions, the median income is lower than the mean income. We say that $\mathbf{x}$ is a *mean-preserving spread* of $\mathbf{y}$ if and only if the variance of $\mathbf{x}$ is strictly higher than the variance of $\mathbf{y}$. We say that $\mathbf{x}$ *median-dominates* $\mathbf{y}$ if and only if the median income under $\mathbf{x}$ is no less than that under $\mathbf{y}$ regardless of other incomes. We say that $\mathbf{x}$ *strictly median-dominates* $\mathbf{y}$ if and only if the median income under $\mathbf{x}$ is strictly higher than that under $\mathbf{y}$ regardless of other incomes. Finally, we say that $\mathbf{x}$ *strongly median-dominates* $\mathbf{y}$ if and only if the two following conditions hold: (i) the income of the median voter under $\mathbf{x}$ is no less than that under $\mathbf{y}$, (ii) the rawlsian criterion under $\mathbf{x}$ is no less than that under $\mathbf{y}$, and at least one of these two inequalities is strict. Note that as the distributions we are considering here have the same mean, the utilitarian criterion is also the same under all of them. Formally, we have the following definition:

Definition 1: Consider the set of vectors

$$\mathbf{X} = \left\{ \mathbf{x} : 0 < x_1 < x_2 < ... < x_n,\ \frac{1}{n}\sum_{i=1}^{n} x_i = \mu \text{ and } x_m < \mu \right\}$$

Let $\mathbf{x}$, $\mathbf{y} \in \mathbf{X}$.

(a) If $var(\mathbf{x}) > var(\mathbf{y})$, we say that $\mathbf{x}$ is a mean-preserving spread of $\mathbf{y}$

(b) If $x_m \geq y_m$, we say that $\mathbf{x}$ *median-dominates* $\mathbf{y}$. If the inequality is strict, we say that $\mathbf{x}$ *strictly median-dominates* $\mathbf{y}$.

(c) If $x_m \geq y_m$ and $\min\{x_1, ..., x_n\} \geq \min\{y_1, ..., y_n\}$, and at least one of these two inequalities is strict, we say that $\mathbf{x}$ *strongly median-dominates* $\mathbf{y}$

Example 1 below illustrates the potential cases arising from this definition:

Example 1: Consider the four income vectors

$$\mathbf{w} = \{0.2, 0.25, 0.75\}$$

$$\mathbf{x} = \{0.1, 0.3, 0.8\}$$

$$\mathbf{y} = \{0.2, 0.3, 0.7\}$$

$$\mathbf{z} = \{0.3, 0.35, 0.55\}$$

Note that for the four sets, the mean is equal and higher than the median:

$$\frac{1}{3}\sum_{i=1}^{3} w_i = \frac{1}{3}\sum_{i=1}^{3} x_i = \frac{1}{3}\sum_{i=1}^{3} y_i = \frac{1}{3}\sum_{i=1}^{3} z_i = 0.4$$

$$w_m = 0.25 < 0.4,\ x_m = 0.3 < 0.4,\ y_m = 0.3 < 0.4,\ z_m = 0.35 < 0.4$$

We also have

$$var(\mathbf{w}) = 0.09,\ var(\mathbf{x}) = 0.13,\ var(\mathbf{y}) = 0.07,\ var(\mathbf{z}) = 0.02$$

$$\Leftrightarrow var(\mathbf{x}) > var(\mathbf{w}) > var(\mathbf{y}) > var(\mathbf{z})$$

then,

(i) $\mathbf{z}$ median-dominates $\mathbf{w}$, $\mathbf{x}$, $\mathbf{y}$, strictly and strongly median-dominates $\mathbf{w}$, $\mathbf{x}$, $\mathbf{y}$

(ii) $\mathbf{x}$ median-dominates $\mathbf{y}$, but does not strictly nor strongly median-dominates $\mathbf{y}$

(iii) $\mathbf{x}$ striclty median-dominates $\mathbf{w}$, but does not strongly median-dominates $\mathbf{w}$

(iv) $\mathbf{y}$ strongly median-dominates $\mathbf{x}$, but does not strictly median-dominates $\mathbf{x}$

(v) $\mathbf{x}$ strictly median-dominates $\mathbf{w}$ and $\mathbf{x}$ is a mean-preserving spread of $\mathbf{w}$

(vi) $\mathbf{z}$ strictly median-dominates $\mathbf{w}$ and $\mathbf{w}$ is a mean-preserving spread of $\mathbf{z}$

As can be seen in example 1, the definition of inequality is not trivial: first, if $\mathbf{x}$ is a mean-preserving spread of $\mathbf{y}$, it means that $\mathbf{x}$ has a bigger variance than $\mathbf{y}$. Hence, $\mathbf{x}$ is more unequal than $\mathbf{y}$. Second, if $\mathbf{x}$ strictly median-dominates $\mathbf{y}$, it means that $\mathbf{x}$ is less skewed than $\mathbf{y}$. Hence, $\mathbf{x}$ is less unequal than $\mathbf{y}$. Third, if $\mathbf{x}$ strongly median-dominates $\mathbf{y}$, it means that $\mathbf{x}$ is less skewed than $\mathbf{y}$ or that the poorest individual is richer under $\mathbf{x}$ than under $\mathbf{y}$, or both. Hence, $\mathbf{x}$ is even less unequal than $\mathbf{y}$ (in the rawlsian sense). However, we see from (v) and (vi) that $\mathbf{x}$ being a mean-preserving spread of $\mathbf{y}$ (i.e. $\mathbf{x}$ more unequal) does not necessarily imply that $\mathbf{y}$ strictly median-dominates $\mathbf{x}$ (i.e. $\mathbf{x}$ more unequal). The same applies to strong median-dominance[7].

Proposition 3 below states that, in our model, the redistribution policy implemented through direct democracy results in lower post-tax income inequality (in the three senses) relative to that measured by pre-tax inequality.

Proposition 3: *At any tax rate $t < 1$,*

(a) the post-tax income vector $\mathbf{c}$ strictly and strongly median-dominates the pre-tax income vector $\mathbf{y}$

(b) the pre-tax income vector $\mathbf{y}$ is a mean-preserving spread of the post-tax income vector $\mathbf{c}$

Proof: in appendix.

In Proposition 4 below, we order the Condorcet winner tax rates in both the fair and selfish economies under different income distributions

Proposition 4: *Let $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{X}$ be two income vectors. Let t_x^S and t_y^S be the selfish Condorcet winner tax rates associated with $\mathbf{x}$ and $\mathbf{y}$ respectively. Similarly, let t_x^F and t_y^F be the fair Condorcet winner tax rates associated with $\mathbf{x}$ and $\mathbf{y}$ respectively. Then,*

(a) If $\mathbf{x}$ strictly median-dominates $\mathbf{y}$, then $t_x^S < t_y^S$ and either $t_x^F < t_y^F$ or $t_y^F < t_x^F$

(b) If $\mathbf{x}$ strongly median-dominates $\mathbf{y}$, then $t_x^S \leq t_y^S$ and $t_x^F < t_y^F$

(c) If $\mathbf{x}$ is a mean-preserving spread of $\mathbf{y}$, then either $t_y^S < t_x^S$ or $t_x^S < t_y^S$ and either $t_y^F < t_x^F$ or $t_x^F < t_y^F$

Proof: in appendix.

To summarize, in the selfish economy, a variation in income inequality is relevant insofar as it concerns the relative position of the middle class (the median income type). A mean-preserving spread leaving the median income unaffected has no effect on the equilibrium level of redistribution. In other words, redistribution in this case does not depend on the variance of the distribution as long as the mean and median incomes remain the same. In the fair economy, a mean-preserving spread leaving the median income unaffected has an effect insofar as the poorest individual's income varies as a consequence of the spread. From (c), we see that higher inequality in the sense of strong median-dominance implies higher redistribution. However, this is not necessarily true for the cases of increased inequality as measured in (a) and (c), depending on the ordering of the lowest and/or median incomes in both distributions. Finally, when the rich gets richer, the equilibrium level of redistribution increases in both economies,

[7] It can be easily shown with $n > 3$ that $\mathbf{x}$ being a mean-preserving spread of $\mathbf{y}$ does not imply that $\mathbf{y}$ strongly median-dominates $\mathbf{x}$ and vice-versa (take $\mathbf{x} = \{1.1, 1.2, 3.1, 4.3, 5.3\}$ and $\mathbf{y} = \{1, 2, 3, 4, 5\}$ for example).

whether selfish or fair. Similarly, an increase in poverty, apart from the poorest individual, reduces the equilibrium tax rate in both economies.

5. Heterogeneous preferences in a three-income-classes economy

Experimental evidence indicates that people are heterogeneous with respect to fairness preferences. If this were not the case, it would be hard to explain why people manage to cooperate in some situations even though it is a dominant strategy for a selfish person not to do so, while in other situations fairness concerns or the desire to cooperate do not seem to play a role (Fehr and Schmidt (2006)). Hence, some people might be more altruistic than others, some are simply totally self-interested, and it might also well be the case that people derive utility from different types of altruism.

According to several experiments, roughly 50 percent of the population is purely selfish. Hence, an important issue is to determine what would be the equilibrium tax rate resulting from the interaction of fair and selfish voters in the economy. In order to do so, we again follow Dhami and al-Nowaihi (2007) who conducted this analysis using self-centered inequity aversion. Fundamentally, assuming a mixture of fair and selfish voters affects the resulting equilibrium tax rate if and only if the identity of the median voter is altered as compared to the case of homogeneous - selfish or fair - preferences.

In this part, we will assume that there are three income classes among the population of voters: low-income, middle/median-income and high-income. Furthermore, to keep the analysis simple, we will concentrate on a 3-voter economy ($n = 3$). Note that we can interpret those three voters as being representative voters of each income-class in the n-voter economy. We still make the assumption that the median income is smaller than the average income, that is, $y_m = y_2 < \overline{y}$. In part 5.1, we will assume that there is full heterogeneity as the fairness parameters can be different across voters belonging to the same income-class. In part 5.2, in order to derive stronger results, we will assume instead that voters might be either selfish or fair. Hence, there is intra-group homogeneity within the groups of fair and selfish voters but inter-group heterogeneity across the two groups. Furthermore, note that in both cases, there is still heterogeneity regarding the level of skill of an individual (ω_i, $i = 1, 2, 3$).

5.1. The general case

Suppose that there is heterogeneity among voters in the sense that the fairness parameters are different across the voters (i.e. λ_i, δ_i, $i = 1, 2, 3$). The question we would like to answer is whether the introduction of heterogeneity among the voters will affect the equilibrium. In other words, we would like to know whether the identity of the median voter will be altered as compared to the case of homogeneity. When $\lambda_i = \lambda$ and $\delta_i = \delta$ for all i, we know that $t_i > t_j$ for $i < j$ as the preferred tax rate of an individual is a strictly decreasing function of his income. Hence, the Condorcet winner tax rate corresponds to the tax rate chosen by the median voter, being the individual with the median ability or income. In the case of the three-voting-classes considered here, the Condorcet winner tax rate with fairness homogeneity will be t_2 as we have $t_3 < t_2 < t_1$. However, this will no longer be necessarily true when we allow for fairness heterogeneity.

The preferred tax rate for each income-class is given by

$$t_1 = \frac{(1-\lambda_1)(\overline{y}-y_1)+\lambda_1\delta_1(\overline{y}-y_1)}{(1-\lambda_1)(2\overline{y}-y_1)+\lambda_1\delta_1(\overline{y}-y_1)+\lambda_1\overline{y}} > 0$$

$$t_2 = \frac{(1-\lambda_2)(\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)}{(1-\lambda_2)(2\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)+\lambda_2\overline{y}} > 0$$

$$t_3 = \frac{(1-\lambda_3)(\overline{y}-y_3)+\lambda_3\delta_3(\overline{y}-y_1)}{(1-\lambda_3)(2\overline{y}-y_3)+\lambda_3\delta_3(\overline{y}-y_1)+\lambda_3\overline{y}} > 0 \text{ if and only if } \lambda_3 > \frac{(\overline{y}-y_3)}{(\overline{y}-y_3)+\delta_3(y_1-\overline{y})}$$

Proposition 5: *Define the following three constants:*

$$\alpha_1 = \overline{y}(\lambda_1\delta_1-\lambda_1-\lambda_2\delta_2+\lambda_2)+y_1(\lambda_2\delta_2-1+\lambda_1-\lambda_1\delta_1)+y_2(1-\lambda_2)$$

$$\alpha_2 = \overline{y}(\lambda_1\delta_1-\lambda_1-\lambda_3\delta_3+\lambda_3)+y_1(\lambda_3\delta_3-1+\lambda_1-\lambda_1\delta_1)+y_3(1-\lambda_3)$$

$$\alpha_3 = \overline{y}(\lambda_2\delta_2-\lambda_2-\lambda_3\delta_3+\lambda_3)+y_1(\lambda_3\delta_3-\lambda_2\delta_2)+y_2(\lambda_2-1)+y_3(1-\lambda_3)$$

Then,
(a) $t_1 > t_2$ *if and only if* $\alpha_1 > 0$
(b) $t_1 > t_3$ *if and only if* $\alpha_2 > 0$
(c) $t_2 > t_3$ *if and only if* $\alpha_3 > 0$
Proof: in appendix.

Note that $t_3 > 0$ if and only if $\lambda_3 > \frac{(\overline{y}-y_3)}{(\overline{y}-y_3)+\delta_3(y_1-\overline{y})}$. If this condition is not satisfied, we will have that $t_3 = 0$. In this case, if $\alpha_3 < 0$ is also satisfied, we would also have $t_2 = 0$. But this is impossible as t_2 is strictly positive. Hence, the two inequalities cannot be simultaneously satisfied.

Let focus on the case where $t_3 > 0$ and thus the fairness parameters of voter 3, λ_3 and δ_3, are such that $\lambda_3 > \frac{(\overline{y}-y_3)}{(\overline{y}-y_3)+\delta_3(y_1-\overline{y})}$. As can be seen from the definition of the three constants above, many cases are potentially possible, depending on the values taken by y_i, λ_i and δ_i for $i = 1, 2, 3$. Indeed, we could have all the following cases:
(a) $\alpha_i > 0$, $i = 1, 2, 3 \Rightarrow t_3 < t_2 < t_1$
(b) $\alpha_1 < 0$, $\alpha_2 > 0$ and $\alpha_3 > 0 \Rightarrow t_3 < t_1 < t_2$
(c) $\alpha_1 > 0$, $\alpha_2 < 0$ and $\alpha_3 < 0 \Rightarrow t_2 < t_1 < t_3$
(d) $\alpha_1 < 0$, $\alpha_2 > 0$ and $\alpha_3 < 0 \Rightarrow t_2 < t_3 < t_1$
(e) $\alpha_1 < 0$, $\alpha_2 < 0$ and $\alpha_3 > 0 \Rightarrow t_1 < t_3 < t_2$
(f) $\alpha_i < 0$, $i = 1, 2, 3 \Rightarrow t_1 < t_2 < t_3$

As we can see from the 6 cases above, many configurations are possible and the decisive voter will not necessarily be the median-income voter any longer. Hence, the introduction of fairness heterogeneity among the population of voters does possibly alter the equilibrium level of redistribution. The example below illustrates cases (a), (b) and (d) respectively.

Example 2: Let $\mathbf{y} = \{0.2, 0.3, 0.7\}$ and thus $\overline{y} = 0.4$. Consider the following cases regarding the value of the fairness parameters:

(i) $\boldsymbol{\lambda} = \{0.2, 0.3, 0.4\}$ and $\delta_i = 0.5$ for $i = 1,\ 2,\ 3$. In this case, $\mathbf{t} = \{0.31, 0.2, 0\}$ and the Condorcet winner tax rate is t_2

(ii) $\lambda_i = 0.5$ for $i = 1,\ 2,\ 3$ and $\boldsymbol{\delta} = \{0.1, 0.8, 0.9\}$. In this case, $\mathbf{t} = \{0.21, 0.24, 0\}$ and the Condorcet winner tax rate is t_1

(iii) $\boldsymbol{\lambda} = \{0.1, 0.1, 0.9\}$ and $\delta_i = 0.9$ for $i = 1,\ 2,\ 3$. In this case, $\mathbf{t} = \{0.33, 0.21, 0.25\}$ and the Condorcet winner tax rate is t_3

In case (i), the weight given to fairness considerations, λ_i, increases with the level of income. In other words, it is the richest individual that cares the most about fairness. Furthermore, the utilitarian and rawlsian criteria are given the same weight for the three voters. In this case, the decisive voter remains the median-skill individual, voter 2. Note that, even though voter 3 does care about fairness, he does not care "enough" about it so as to choose a positive tax rate. In case (ii), voter 1 gives a lot of weight to the utilitarian criterion in his utility function relatively to case (i), which makes him vote for a much lower tax rate. Voter 2, in contrast, will vote for a higher tax rate than in case (i) as he cares relatively more about the poorest individual. As a result, the order between the two tax rates is reversed. Finally, in case (iii), voter 3 derives a huge utility from giving away resources to the poorest individual (both his fairness parameters are very high). Hence, his preferred tax rate is positive and even higher than the one chosen by voter 2.

5.2. The fair versus selfish case

As the general case gives rise to many possibilities, we will now assume that voters are either selfish or fair, and that the fair voters share the same fairness parameters, that is,

$$\lambda_i = \lambda \text{ and } \delta_i = \delta \text{ for } i = 1,\ 2,\ 3$$

For the selfish voters, $\lambda_i = \delta_i = 0$ for $i = 1,\ 2,\ 3$. For the n-voter economy, there are 2^n cases to consider. Here, with three voters, there are only $2^3 = 8$ cases to consider. Let S and F be a selfish and a fair voter respectively. The 8 combinations of voters, by increasing income level, are the following: SSS, FFF, SFF, SFS, FSF, FFS, SSF, FSS. The first two cases are the cases of homogeneity. Hence, we concentrate now on the remaining 6 cases.

The preferred tax rates of a fair voter with the three levels of income are given by the same expressions as in (4.1.) where $\lambda_i = \lambda$ and $\delta_i = \delta$ for $i = 1$, 2, 3. Similarly, the preferred tax rates of a selfish voter with the three levels of income are given by

$$t_1^S = \frac{\overline{y} - y_1}{2\overline{y} - y_1} > 0$$

$$t_2^S = \frac{\overline{y} - y_2}{2\overline{y} - y_2} > 0$$

$$t_3^S = \frac{\overline{y} - y_3}{2\overline{y} - y_3} < 0 \Rightarrow t_3^S = 0^8$$

Proposition 6: *In the three-voter economy where voters might be either selfish or fair,*

(a) A lower income selfish voter prefers a higher tax rate than a higher income selfish voter, that is, $t_3^S < t_2^S < t_1^S$

(b) A lower income fair voter prefers a higher tax rate than a higher income fair voter, that is, $t_3^F < t_2^F < t_1^F$

(c) The low-income selfish voter prefers a higher tax rate than the low-income fair voter, that is, $t_1^F < t_1^S$

(d) The median-income fair voter prefers a higher tax rate than the median-income selfish voter if and only if the fairness parameter associated to the rawlsian criterion, δ*, is high enough, that is,* $t_2^S < t_2^F$ *if and only if* $\delta > \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$

(e) The high-income fair voter prefers a higher tax rate than the high-income selfish voter, and hence prefers a strictly positive tax rate, if and only if the fairness parameters, λ *and* δ*, are such that* $\lambda > \frac{(\overline{y} - y_3)}{(\overline{y} - y_3) + \delta(y_1 - \overline{y})}$*, that is,* $t_3^S < t_3^F$ *if and only if* $\lambda > \frac{(\overline{y} - y_3)}{(\overline{y} - y_3) + \delta(y_1 - \overline{y})}$

Proof: (a) and (b) follow from the fact that the preferred tax rate of an individual is decreasing in his level of income. (c), (d) and (e) follow from direct calculation.

As already said, parts (a) and (b) of Proposition 5 hold because a lower income voter, being selfish or fair, benefits more from a redistributive tax than a higher income voter. Part (c) holds because the low income fair voter, as he values surplus maximization through the utilitarian motive in his preferences, thus prefers a lower tax rate than the low income selfish voter. From parts (d) and (e), we see that a fair voter, whether enjoying middle or high income, does not necessarily vote for a higher tax rate than the corresponding selfish voter. This is not surprising as we saw from Proposition 2b that an increase in the weight associated to the social welfare criterion λ has an ambigous effect on the redistributive tax rate arising from the utilitarian motive in the political preferences. For the median-income voter, we obtain the same restriction as in Proposition 2b in order to have a fair tax rate higher than the corresponding selfish one. For the high-income individual, as the selfish type votes for no redistribution, we have that the fair tax rate is higher than the selfish one, and

[8] Notice that here we have to assume that the distribution of income is such that $y_3 < 2\overline{y}$ so that $t_3^S \in [0, 1]$

hence strictly positive, if and only if the weight associated to fairness, λ, is high enough given the value of δ.

In any combination of the selfish and fair voters, the redistributive outcome is altered relatively to the case of homogeneity of preferences if and only if the identity of the median voter is altered. The results arising from considering the various cases are summarized in Proposition 7 below.

Proposition 7: *In the three-voter economy where voters might be either selfish or fair,*

(a) The median-income voter is decisive, and $t_3 < t_2 < t_1$, in the following cases:

(i) FFS, SFF and SFS

(ii) SSF, if $\lambda < \frac{(y_2 - y_3)}{(\overline{y} - y_3) + \delta(y_1 - \overline{y})}$

(iii) FSS, if $\delta > \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ or if $\delta < \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ and $\lambda(1-\delta) < \frac{y_2 - y_1}{\overline{y} - y_1}$

(iv) FSF, if $\delta > \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ and $\lambda < \frac{(y_2 - y_3)}{(\overline{y} - y_3) + \delta(y_1 - \overline{y})}$ or if $\delta < \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ and $\lambda(1-\delta) < \frac{y_2 - y_1}{\overline{y} - y_1}$

(b) The low-income voter is decisive, and $t_3 < t_1 < t_2$, in cases FSS and FSF, if $\delta < \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ and $\lambda(1-\delta) > \frac{y_2 - y_1}{\overline{y} - y_1}$

(c) The high-income voter is decisive, and $t_2 < t_3 < t_1$, in the following cases:

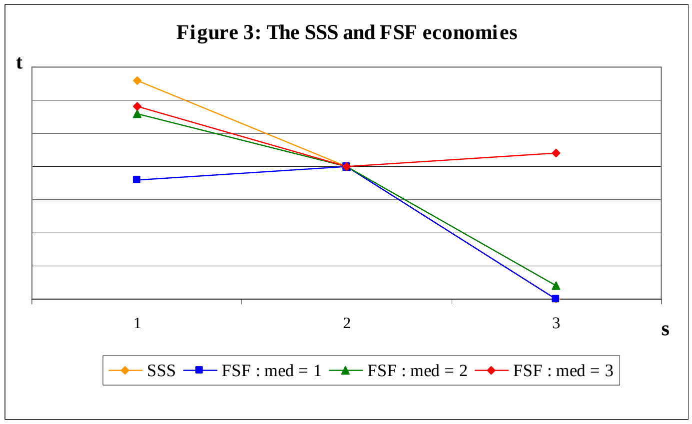
(i) SSF, if $\lambda > \frac{(y_2 - y_3)}{(\overline{y} - y_3) + \delta(y_1 - \overline{y})}$

(ii) FSF, if $\delta > \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ and $\lambda > \frac{(y_2 - y_3)}{(\overline{y} - y_3) + \delta(y_1 - \overline{y})}$

Proof: in appendix.

In cases FFS, SFF and SFS, although there is a mixture of fair and selfish voters in the economy, the decisive power when voting for the redistributive tax rate remains in the hands of the median-income voter, voter 2 (Proposition 7a(i)). Hence, in those cases, the presence of selfish individuals in the economy does not alter the redistributive outcome relatively to the case where all voters are fair. The opposite is true for the cases SSF, FSS and FSF when the appropriate parameter restrictions are satisfied (Proposition 7a(ii), (iii) and (iv)): the median-income sefish voter is decisive, and thus the introduction of fair voters in the economy does not matter for the policy choice on redistribution.

However, it might also be the case that the median-income voter is no longer decisive. In cases FSS and FSF, the low-income voter becomes the decisive voter provided that the corresponding parameter restrictions are satisfied (Proposition 7b). Similarly, in cases SSF and FSF, the possibility arises that the redistributive outcome is controlled by the high-income individual, voter 3. In all cases, the Condorcet winner tax rate coincides with the median tax rate, and not necessarily with the tax rate preferred by the median-income individual.



Before discussing the results in Proposition 7, we would like to highlight the following striking cases: first, in the SFS, SSF and FSS economies, the majority of voters are *selfish*. However, provided that the corresponding parameter conditions are fulfilled (and always in the SFS economy), the redistributive outcome ends up being controlled by a *fair* voter. Second, in the FSF economy, the opposite is true: under specific parameter values, the decisive voter ends up being the *selfish* voter, even though the majority of voters are *fair*.

Using the parameter restrictions specified in Proposition 7, we construct the following example illustrating the various possibilities for the FSF economy:

Example 3: Denote by $\mathbf{t}^S = \left\{t_1^S, t_2^S, t_3^S\right\}$ the vector of preferred tax rates chosen by voter 1, 2 and 3 respectively when they are selfish. Similarly, denote by $\mathbf{t}^F = \left\{t_1^F, t_2^F, t_3^F\right\}$ the vector of preferred tax rates chosen by voter 1, 2 and 3 respectively when they are fair. Let $\mathbf{y} = \{0.2, 0.3, 0.7\}$ and thus $\overline{y} = 0.4$. Hence, $\mathbf{t}^S = \{0.33, 0.2, 0\}$. Consider the following cases:

(i) If $\lambda = \delta = 0.7$, we have $\mathbf{t}^F = \{0.28, 0.24, 0.02\}$. Therefore, in the FSF economy, $t_3 < t_2 < t_1$.

(ii) If $\delta = 0.2$ and $\lambda = 0.7$, we have $\mathbf{t}^F = \{0.18, 0.13, 0\}$. Therefore, in the FSF economy, $t_3 < t_1 < t_2$.

(iii) If $\delta = 0.8$ and $\lambda = 0.9$, we have $\mathbf{t}^F = \{0.29, 0.28, 0.22\}$. Therefore, in the FSF economy, $t_2 < t_3 < t_1$.

In order to discuss the results in Proposition 7, we focus on the FSF economy, as this case reassembles all the possibilities: provided that the corresponding parameter restrictions are satisfied, the decisive voter might be either the low-income, middle-income or high-income voter.

In the SSS economy, the median-income voter is decisive and $t_3 < t_2 < t_1$. Suppose now that the low and high-income voters are fair so as to generate the

FSF economy, and assume furthermore that δ is high, that is, $\delta > \frac{y_2-\overline{y}}{y_1-\overline{y}}$. In our example, this corresponds to $\delta > \frac{1}{2}$. If λ is low given δ, that is, if $\lambda < \frac{(y_2-y_3)}{(\overline{y}-y_3)+\delta(y_1-\overline{y})}$, the median-income voter remains decisive and $t_3 < t_2 < t_1$. If, on the contrary, λ is higher than this threshold, the high-income voter becomes decisive, and $t_2 < t_3 < t_1$. This case is illustrated by the red curve in figure 3, where there is a 10 percent increase in the redistributive tax rate (from t_2^S to t_3^F). However, note that, as can be seen in example 3, for t_3 to be the median tax rate, the value of λ must be really high[9] for a given δ. Hence, even though this case is theoretically possible, it is not very likely to happen in reality. Indeed, if individuals might have a preference for fairness, the majority of them for sure do not put nearly all the weight on fairness considerations relatively to their own self-interest when taking a decision.

Suppose now that δ is low, that is, $\delta < \frac{y_2-\overline{y}}{y_1-\overline{y}}$ ($\delta < \frac{1}{2}$ in the example). Again, if λ is low given δ, that is, if $\lambda(1-\delta) < \frac{y_2-y_1}{\overline{y}-y_1}$, the median-income voter remains decisive, and $t_3 < t_2 < t_1$. If, on the contrary, λ is higher than this threshold, the low-income voter becomes decisive, and $t_3 < t_1 < t_2$. This case is illustrated by the blue curve in figure 3, where there is a 10 percent decrease in the redistributive tax rate (from t_2^S to t_1^F).

What can be said about those paramater restrictions and the implied cases in the context of the FSF economy?

Case 1: if δ is high, that is, $\delta > \frac{y_2-\overline{y}}{y_1-\overline{y}}$, the weight given to the rawlsian motive relatively to the utilitarian motive is high. This fact, independently of the value of λ, contributes to increase the tax rate preferred by a fair voter. Note that this restriction on the parameter δ ensures in fact[10] that $t_2^F > t_2^S$. Then, If λ is low given this high δ, that is, if $\lambda < \frac{(y_2-y_3)}{(\overline{y}-y_3)+\delta(y_1-\overline{y})}$, it means that the weight given to fairness considerations relatively to self-interest is low. Hence, in the FSF economy, it becomes more likely that the high-income voter, despite the fact of caring about fairness, will prefer a lower tax rate than the selfish median-income voter. On the contrary, when λ gets higher, and given the fact that the rawlsian motive is given a lot of weight, the tax rate preferred by the high-income fair voter increases correspondingly. When λ reaches the value of $\frac{(y_2-y_3)}{(\overline{y}-y_3)+\delta(y_1-\overline{y})}$, we will have that $t_3^F > t_2^S$.

Now, what are the factors conducting to satisfy those inequalities, leading to voter 3 being the decisive voter?

(a) As we saw, δ has to be high, whatever the value of λ, so that $t_2^F > t_2^S$. More precisely, δ has to be higher than $\frac{y_2-\overline{y}}{y_1-\overline{y}}$. Hence, the higher the distance

[9] Moreover, δ also has to be high enough for a given income distribution. If δ is too low, t_3 being the median tax rate requires $\lambda > 1$, which cannot happen.

[10] Otherwise, it could not be the case that the tax rate preferred by the high-income voter coincides with the median tax rate. This is also true for the SSF economy (see the proof of Proposition 7 in appendix).

between the median and mean incomes, and the lower the distance between the low and mean incomes, the higher the required δ so as to satisfy $t_2^F > t_2^S$. Suppose that y_1 increases. As a result, $\frac{y_2-\overline{y}}{y_1-\overline{y}}$ increases. The selfish median voter would like more redistribution as average income increases. However, the fair median voter would like less redistribution.

(b) Given the (high) value of δ, we need λ to be high enough, that is, we need $\lambda > \frac{(y_2-y_3)}{(\overline{y}-y_3)+\delta(y_1-\overline{y})}$. Hence, the higher the distance between the median and high incomes (i.e. higher inequality at the upper end of the income distribution), and the lower the distance between the mean and high incomes one the one hand, and the mean and low incomes on the other hand, the higher the required λ so as to satisfy $t_3^F > t_2^S$. Suppose that y_3.increases. As a result, $\frac{(y_2-y_3)}{(\overline{y}-y_3)+\delta(y_1-\overline{y})}$ increases. The selfish voter would like to redistribute more as average income increases and thus the benefits associated to taxation increase. In contrast, the high-income fair voter, speaking about his own income, would like to redistribute less.

(c) Finally, the higher δ, the lower the required λ so as to satisfy $t_3^F > t_2^S$. Indeed, the more weight given to the rawlsian motive, the less "fairness" needed in order to obtain $t_3^F > t_2^S$. Remember that in order for t_3^F to be strictly positive, we need $\lambda > \frac{(\overline{y}-y_3)}{(\overline{y}-y_3)+\delta(y_1-\overline{y})}$. If we want it to be higher than t_2^S, which is itself strictly positive, an even higher λ is required.

Case 2: if δ is low, that is, $\delta < \frac{y_2-\overline{y}}{y_1-\overline{y}}$, we have $t_2^F < t_2^S$. Then, If λ is low given this low δ, that is, if $\lambda(1-\delta) < \frac{y_2-y_1}{\overline{y}-y_1}$, it means that the weight given to fairness considerations relatively to self-interest is low. Hence, in the FSF economy, it becomes more likely that the low-income fair voter will prefer a higher tax rate than the selfish median-income voter (remember that $t_1^F < t_1^S$). On the contrary, when λ gets higher, the tax rate preferred by the low-income fair voter decreases correspondingly. When λ reaches the value of $(1-\delta)^{-1}\left(\frac{y_2-y_1}{\overline{y}-y_1}\right)$, we will have that $t_2^S > t_1^F$.

Now, what are the factors conducting to satisfy those inequalities, leading to voter 1 being the decisive voter?

(a) As we saw, δ has to be low, whatever the value of λ, so that $t_2^F < t_2^S$. More precisely, δ has to be lower than $\frac{y_2-\overline{y}}{y_1-\overline{y}}$. The intuition here is the same as in case 1(a) above.

(b) Given the (low) value of δ, we need λ to be high enough, that is, we need $\lambda > (1-\delta)^{-1}\left(\frac{y_2-y_1}{\overline{y}-y_1}\right)$. Hence, the higher the distance between the median and low incomes (i.e. higher inequality at the lower end of the income distribution), and the lower the distance between the mean and low incomes, the higher the required λ so as to satisfy $t_2^S > t_1^F$. Suppose that y_1 decreases. As a result, $\left(\frac{y_2-y_1}{\overline{y}-y_1}\right)$ increases. The selfish voter would like to redistribute less, whereas the poor fair voter would like to redistribute more as his own income has decreased.

Hence, it will be less likely that $t_2^S > t_1^F$, and thus a higher λ is needed in order to satisfy this inequality.

(c) Finally, the higher δ, the higher the required λ so as to satisfy $t_2^S > t_1^F$. Indeed, the more weight given to the rawlsian motive, the more "fairness" needed in order to obtain $t_2^S > t_1^F$. This makes sense as we know that t_1^F is increasing in δ but decreasing in λ.

6. A comparison with self-centered inequity/inequality aversion (Fehr and Schmidt (1999))

In this part, we will compare our results with the ones obtained when assuming two-sided self-centered inequality aversion as defined by Fehr and Schmidt (1999). This exercise has been carried out by Dhami and al-Nowaihi (2008a and b) using the same baseline model of voting on redistribution. As both the "difference aversion" and "social welfare" models are very discussed and find empirical support in the literature, we would like to know whether they yield different theoretical predictions regarding the extent of redistribution in the context of a direct democracy.

The utility function proposed by Fehr and Schmidt (1999) in order to capture the idea if inequity/inequality aversion is the following:

$$U_i\left(x_1,...,x_n\right) = x_i - \frac{\alpha_i}{n-1}\sum_{j\neq i}\max\left\{x_j - x_i, 0\right\} - \frac{\beta_i}{n-1}\sum_{j\neq i}\max\left\{x_i - x_j, 0\right\}$$

with $0 \leq \beta_i \leq \alpha_i$ and $\beta_i \leq 1$. Note that $\frac{\partial U_i}{\partial x_j} > 0$ if and only if $x_i \geq x_j$. Note also that the disutility from inequality is larger if another person is better off than player i than if another person is worse off. Finally, note that envy is unbounded.

6.1. The tax rate and comparative statics

Dhami and al-Nowaihi (2008a and b) compute the equilibrium redistributive tax rate using inequity-averse political preferences. More specifically, the voters choose their preferred tax rate by maximizing their indirect utility function, given by:

$$\begin{aligned} V_i\left(t,b,\omega_i\right) &= \upsilon\left(t,b,\omega_i\right) - \frac{\alpha}{n-1}\sum_{k\neq i}\max\left\{0,\upsilon\left(t,b,\omega_k\right) - \upsilon\left(t,b,\omega_i\right)\right\} \\ &\quad -\frac{\beta}{n-1}\sum_{i\neq j}\max\left\{0,\upsilon\left(t,b,\omega_j\right) - \upsilon\left(t,b,\omega_i\right)\right\} \end{aligned}$$

The median voter's preferred tax rate[11] under self-centered inequality aver-

[11] The authors show that this function satisfies the single-crossing property, which guarantees the existence of an equilibrium. As for the case of single-peakedness, the Condorcet winner tax rate is the one chosen by the median skill voter.

sion is given by

$$t_m = \frac{\overline{y} - y_m + \frac{\alpha}{n-1}\sum_{k>m}(y_k - y_m) + \frac{\beta}{n-1}\sum_{i<m}(y_m - y_i)}{2\overline{y} - y_m + \frac{\alpha}{n-1}\sum_{k>m}(y_k - y_m) + \frac{\beta}{n-1}\sum_{i<m}(y_m - y_i)}$$

Note that the Condorcet winner tax rate when voters are inequity averse depends on the whole income distribution, whereas it only depends on the mean, median and lowest incomes under quasi-maximin altruism.

The comparative statics results obtained with self-centered inequity aversion of this type are different from the ones we obtained with social preferences of the quasi-maximin type. First, while in our set up, an increase in the parameter λ induces more redistribution only when δ is high, an increase in both the parameters α and β results unambiguously in higher redistribution with self-centered inequity aversion. An increase in α increases disutility arising from disadvantageous inequity (envy). Hence, by increasing the tax rate, the median voter reduces this disutility as it reduces relatively the utility of anyone who is richer. On the other hand, an increase in β increases disutility arising from advantageous inequity (altruism). Hence, increasing the tax rate benefits everyone poorer than the median voter relatively more, thereby reducing advantageous inequity. As a result, with self-centered inequity aversion, a fair median voter always chooses a higher tax rate than a selfish median voter. In contrast, remember that, in our set up, increasing alstruism (i.e. λ) has two effects. On the one hand, it increases the equilibrium tax rate which comes from the altruistic benefit of redistributing to the poor. On the other hand, the altruistic - and efficiency - cost of taxing the rich decreases the equilibrium tax rate. Which effects dominates depends on altruistic preferences towards the rich and the poor (and thus on the parameter δ), and on the income distribution.

The other difference arising from using inequity aversion relates to the change in the equilibrium tax rate following an increase in y_j for $j < m$ (i.e. an increase in any below-median income, or, in other words, a decrease in poverty). While in our set up, an increase in *any* voter's income except from the poorest one induces an increase in redistribution, this is not the case with self-centered inequity aversion. Indeed, Dhami and al-Nowaihi (2008b) show that, when α or β are "high enough", an increase in poverty (i.e. a decrease in y_j for $j < m$) will increase the tax rate chosen by the median voter. The intuition is the following: on the one hand, the inequity-averse voter cares about his own payoff, which induces him to decrease redistribution following a decrease in y_j for all j. On the other hand, the concern for poorer voters arising from advantageous inequity induces him in the opposite direction, i.e. towards greater redistribution. When α or/and β are sufficiently high, this second effect becomes stronger, yielding to higher redistribution following an increase in poverty. With quasi-maximin preferences, this effect exists only as far as the poorest voter is concerned. In contrast, when y_j decreases for any $j < m$, $j \neq 1$, the average income available for redistribution decreases, thereby reducing the marginal benefits of increasing the tax rate (as in the standard model with selfish voters).

6.2. The link between fairness, inequality and redistribution

In this part, we will check whether assuming inequity aversion instead of quasi-maximin altruism yields different theoretical predicitons regarding the relation between inequality and redistribution.

When voters are inequity averse and the rich gets richer, the level of redistribution increases, as for the case of quasi-maximin altruism. Inequity averse voters have an additional motive to increase redistribution when the rich gets richer: as the selfish voters, they do so because of the corresponding increase in average income. However, they are also motivated by the desire to reduce disadvantageous inequity.

When the poor gets poorer, we saw that the selfish median voter votes for a lower tax rate. Again, this is so because the increased poverty reduces average income available for redistribution. When the median voter has quasi-maximin preferences, this is also true except in the case of the poorest voter getting poorer. In this case, and only in this case, the redistributive tax rate increases in response to an increase in poverty. In contrast, and as we just explained in the comparative statics comparison, Dhami and al-Nowaihi (2008b) show that, when the median voter is inequity averse, an increase in poverty (i.e. a decrease in *any* below-median income) will result in a higher level of redistribution provided that the fairness parameters are high enough.

Assuming that the rise in inequality leaves the mean unaffected, we saw that only two things matter regarding the link between inequality and redistribution under quasi-maximin altruism: the relative position of the median and lowest incomes after the rise in inequality. If a distribution $\mathbf{x}$ strongly median-dominates another distribution $\mathbf{y}$, and only in this case, we will have for sure that the redistributive tax rate under $\mathbf{x}$ is smaller than under $\mathbf{y}$ (i.e. a positive link between inequality and redistribution). Otherwise, a mean-preserving spread of the income distribution has an ambiguous effect on the level of redistribution.

When voters are inequity averse, the whole income distribution matters for the choice of the optimal level of redistribution. Therefore, a mean-preserving spread also has an ambiguous effect on the redistributive outcome, depending on the resulting advantageous and disadvantageous inequity for the decisive voter. In the context of inequity aversion, Dhami and al-Nowaihi (2008b) define the concept of strong median-dominance in the following way: *let* $\mathbf{x}$ *and* $\mathbf{y} \in \mathbf{X}$ *be two income distributions. Suppose* $x_m \geq y_m$, $\sum_{k>m}(x_k - x_m) \leq \sum_{k>m}(y_k - y_m)$, $\sum_{i<m}(x_m - x_i) \leq \sum_{i<m}(y_m - y_i)$ *and, at least, one of these inequalities is trict. Then* $\mathbf{x}$ *strongly median-dominates* $\mathbf{y}$. In other words, a distribution $\mathbf{x}$ strongly median-dominates a distribution $\mathbf{y}$ if and only if $\mathbf{x}$ is

no more skewed than $\mathbf{y}$, and both advantageous and disadvantageous inequity are not higher under $\mathbf{x}$ than under $\mathbf{y}$ (with at least one strict inequality). The authors show that in this case, and only in this case, we will have for sure that the redistributive tax rate under $\mathbf{x}$ will be smaller than under $\mathbf{y}$ (i.e. also a positive link between inequality and redistribution).

6.3. The mixture of fair and selfish voters in the three-income-classes economy

The relationship we found between the selfish and fair tax rates for each income level turns out to be different from the one obtained assuming self-centered inequity aversion. While Dhami and al-Nowaihi (2008a) also find a strictly negative relationship between the preferred tax rate of a voter (whether sefish or fair) and his ability (and hence, his income), they find that the fair tax rate cannot be smaller than the selfish one whatever the income level, that is, $t_i^F \geq t_i^S$, $i = 1, 2, 3$. This is so because, in addition to the usual selfish reasons to desire a positive level of redistribution, fair voters care about lower-skill voters and envy higher-skill voters.

However, the relationships we found in our set up are not surprising as we saw from Proposition 2b that an increase in the weight associated to the social welfare criterion λ has an ambiguous effect on the redistributive tax rate arising from the utilitarian motive in the political preferences. Hence, as we saw, the low-income selfish voter will vote for a higher tax rate than the low-income fair voter as the latter cares for the maximization of the surplus, and thus wants to minimize distorsions in the economy. For voter 2, we obtained the same restriction as in Proposition 2b in order to have a fair tax rate higher than the corresponding selfish one (δ has to be high). For the high-income voter, as the selfish type votes for no redistribution, we had that the fair tax rate is higher than the selfish one, and hence strictly positive, if and only if the weight associated to fairness, λ, is high enough given δ.

We found several cases for which the identity of the median voter alters as a result of heterogeneity. The table below summarizes the results:

Economy	**Ranking**	**Restrictions**
SSF	$t_2^S < t_3^F < t_1^S$	$\lambda > \frac{(y_2 - y_3)}{(\overline{y} - y_3) + \delta(y_1 - \overline{y})}$
FSF	$t_2^S < t_3^F < t_1^F$ $t_3^F < t_1^F < t_2^S$	$\delta > \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ and $\lambda > \frac{(y_2 - y_3)}{(\overline{y} - y_3) + \delta(y_1 - \overline{y})}$ $\delta < \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ and $\lambda(1 - \delta) > \frac{y_2 - y_1}{\overline{y} - y_1}$
FSS	$t_3^S < t_1^F < t_2^S$	$\delta < \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$ and $\lambda(1 - \delta) > \frac{y_2 - y_1}{\overline{y} - y_1}$

For the case of self-centered inequity aversion, Dhami and al-Nowaihi (2008a) identify the following cases:

Economy	**Ranking**	**Restrictions**
SFS	$t_3^S < t_1^S < t_2^F$	$\frac{\alpha}{2-\beta} > \frac{y_2-y_1}{y_3-y_2}$
SFF	$t_3^F < t_1^S < t_2^F$	$\frac{\alpha}{2-\beta} > \frac{y_2-y_1}{y_3-y_2}$
SSF	$t_2^S < t_3^F < t_1^S$	$\beta > \frac{2(y_3-y_2)}{2y_3-y_2-y_1}$
FSF	$t_2^S < t_3^F < t_1^F$	$\beta > \frac{2(y_3-y_2)}{2y_3-y_2-y_1}$

To start with, we note some similarities arising from the two tables: first, for the two types of fairness, the high-income fair individual turns out to be the decisive voter in the SSF economy provided that the relevant fairness parameter is high enough. Second, in the FSF economy, this is also true provided this time that both fairness parameters are high in the case of quasi-maximin altruism. Note that in the case of quasi-maximin altruism, the low-income fair voter can also become the decisive voter in the FSF economy, which can never happen in the case of inequity aversion.

Several differences also appear from comparing the two tables. First, the cases for which the identity of the median voter might be altered are not the same for the two types of other-regarding preferences. While in our set up, this possibility could arise in the FSS economy, Dhami and al-Nowaihi (2008a) show that, with inequity aversion, the identity of the median voter might change in the SFS and SFF economies. Second, in all the cases we identified for which the decisive voter and the median-income voter do not coincide, it turns out that the decisive power goes to a fair voter. As we saw, this is most striking in the cases SSF and FSS where two thirds of the population is selfish, as it implies that the redistributive outcome might end up being controlled by a minority of fair voters. Hence, in those cases, the presence of fair voters, even in minority, might have large effects on redistribution policy. This has also been pointed out by Tyran and Sausgruber (2006, p. 470), whose model predicts "a lot of redistribution [assuming] only "a little fairness", i.e. [assuming] that people are at most as fairness-minded as is empirically plausible". In contrast, with inequity aversion, if the low-income voter turns out to be decisive, he must be on the selfish type. In the SFF economy, this means that, even though the majority of voters are fair, the redistribution policy is chosen by the minority of poor and selfish voters.

7. Conclusion

We endowed individuals with quasi-maximin political preferences in a standard model of voting on redistribution in a direct democracy and we showed that the Condorcet winner tax rate with fairness is not necessarily higher than the one resulting from an economy with selfish voters. In this set tup, an increase in poverty increases redistribution insofar as it concerns the poorest individual, provided that the fairness parameters are high enough. More generally, an increase in inequality matters for the equilibrium as long as the median, mean and/or lowest incomes are affected. This contrasts with the results obtained assuming self-centered inequity aversion, as the whole income distribution matters for the equilibrium in this case. Furthermore, with inequity aversion, the Condorcet winner tax rate is unambiguously higher than the one of the selfish economy.

Assuming a mixture of fair and selfish voters in a three-voter economy, we identified three cases in which the identity of the median voter and the median skill voter might diverge, provided that appropriate parameter restrictions are satisfied. In all the three cases, and even when two thirds of the voters are selfish, the redistributive outcome ends up being controled by a fair voter. Interestingly, introducing poor fair voters in a selfish economy might decrease the equilibrium level of redistribution. Then, introducing selfish middle-class voters in a fair economy might either increase or decrease the equilibrium level of redistribution. Finally, introducing either rich or poor selfish voters in a fair economy will have no effect.

8. References

1. Acemoglu, D. and Robinson, J. A. (2005), Economic Origins of Dictatorship and Democracy, Cambridge University Press, New York.

2. Alesina, A., Glaeser, E. and Sacerdote, B. (2001), Why Doesn't the United States Have a European-Style Welfare State, *Brookings Papers on Economic Activity*, Issue 2, pp. 187-277.

3. Benabou, R. and Ok, E. A. (2001), Social Mobility and the Demand for Redistribution: the POUM Hypothesis, *Quarterly Journal of Economics*, Vol. 116, No. 2, pp. 447-487.

4. Borck, R. (2007), Voting, Inequality and Redistribution, *Journal of Economic Surveys*, Vol. 21, No. 1, pp. 90-109.

5. Charness, G and Rabin, M. (2002), Understanding Social Preferences with Simple Tests, *Quarterly Journal of Economics*, Vol. 117, No. 3, pp. 817-869.

6. Dhami, S. and al-Nowaihi, A. (2008a), Social Preferences and Redistribution under Direct Democracy, Working Paper No. 08/11, University of Leicester.

7. Dhami, S. and al-Nowaihi, A. (2008b), Inequality and Size of the Government when Voters Have Other-Regarding Preferences, Working Paper No. 08/23, University of Leicester.

8. Engelmann, D.and Strobel, M. (2004), Inequality Aversion, Efficiency, and Maximin Preferences in Simple Distribution Experiments, *American Economic Review*, Vol. 94, No. 4, pp. 857-869.

9. Fehr, E. and Schmidt, K. (1999), A Theory of Fairness, Competition, and Cooperation, *Quarterly Journal of Economics*, Vol. 114, No. 3, pp. 817-868.

10. Fehr, E. and Schmidt, K. (2006), The Economics of Reciprocity, Fairness and Altruism - Experimental Evidence and New Theories, in Handbook of the Economics of Giving, Altruism and Reciprocity, pp. 615-692, North-Holland.

11. Galasso, V. (2003), Redistribution and Fairness: A Note, *European Journal of Political Economy*, Vol. 19, No. 4, pp 885-892.

12. Meltzer, A. H. and Richard, S.F. (1981), A Rational Theory of the Size of Government, *Journal of Political Economy*, Vol. 89, No. 5, pp. 914-927.

13. Milanovic, B. (2000), The median-Voter Hypothesis, Income Inequality, and Income Redistribution: An Empirical Test with the Required Data, *European Journal of Political Economy*, Vol. 16, Issue 3, pp. 367-410.

14. Roberts (1977), Voting over Income Tax Schedules, *Journal of Public Economics*, Vol. 8, Issue 3, pp. 329-340.

15. Romer (1975), Individual Welfare, Majority Voting, and the Properties of a Linear Income Tax, *Journal of Public Economics*, Vol. 4, Issue 2, pp. 163-185.

16. Tyran, J-R. and Sausgruber, R. (2006), A Little Fairness May Induce a Lot of Redistribution in Democracy, *European Economic Review,* Vol. 50, No. 2, pp. 469-485.

APPENDIX

Proof of Proposition 1:

(a) Substituting for the expression of $\upsilon_i = \upsilon(t, \omega_i)$ into $V_i(\upsilon_1, ..., \upsilon_n)$ yields

$$\begin{aligned} V_i(t,\omega_i) &= (1-\lambda)\left[\frac{1}{2}(1-t)^2\omega_i^2 + \frac{t(1-t)}{n}\sum_{i=1}^{n}\omega_i^2\right] \\ &+\lambda\left\{\begin{array}{c} \delta\left[\frac{1}{2}(1-t)^2\omega_1^2 + \frac{t(1-t)}{n}\sum_{i=1}^{n}\omega_i^2\right] \\ +(1-\delta)\left[\frac{1}{2}(1-t)^2\frac{1}{n}\sum_{i=1}^{n}\omega_i^2 + \frac{t(1-t)}{n}\sum_{i=1}^{n}\omega_i^2\right] \end{array}\right\} \end{aligned}$$

Taking partial derivative with respect to the tax rate t yields

$$\begin{aligned} \frac{\partial V_i(t,\omega_i)}{\partial t} &= -(1-\lambda)\omega_i^2(1-t) + \frac{1}{n}(1-\lambda)(1-2t)\sum_{i=1}^{n}\omega_i^2 - \lambda\delta\omega_1^2(1-t) \\ &+\frac{1}{n}\lambda\delta(1-2t)\sum_{i=1}^{n}\omega_i^2 - \lambda(1-\delta)(1-t)\frac{1}{n}\sum_{i=1}^{n}\omega_i^2 \\ &+\lambda(1-\delta)(1-2t)\frac{1}{n}\sum_{i=1}^{n}\omega_i^2 \end{aligned}$$

Setting this quantity equal to zero and solving for t yields

$$t(\omega_i) = \frac{(1-\lambda)\left(\frac{1}{n}\sum\limits_{i=1}^{n}\omega_i^2 - \omega_i^2\right) + \lambda\delta\left(\frac{1}{n}\sum\limits_{i=1}^{n}\omega_i^2 - \omega_1^2\right)}{(1-\lambda)\left(\frac{2}{n}\sum\limits_{i=1}^{n}\omega_i^2 - \omega_i^2\right) + \lambda\delta\left(\frac{1}{n}\sum\limits_{i=1}^{n}\omega_i^2 - \omega_1^2\right) + \lambda\frac{1}{n}\sum\limits_{i=1}^{n}\omega_i^2} \in (0,1)$$

The second derivative of $V_i(t,\omega_i)$ with respect to t is given by

$$\begin{aligned} \frac{\partial^2 V_i(t,\omega_i)}{\partial t^2} &= (1-\lambda)\omega_i^2 - \frac{2}{n}(1-\lambda)\sum_{i=1}^{n}\omega_i^2 + \lambda\delta\omega_1^2 \\ &-\frac{2}{n}\lambda\delta\sum_{i=1}^{n}\omega_i^2 - \lambda(1-\delta)\frac{1}{n}\sum_{i=1}^{n}\omega_i^2 \\ &= (1-\lambda)y_i - (1-\lambda)2\overline{y} + \lambda\delta y_1 - \lambda\delta\overline{y} - \lambda(1-\delta)\overline{y} \\ &= (1-\lambda)(y_i - 2\overline{y}) + \lambda\delta(y_1 - \overline{y}) - \lambda(1-\delta)\overline{y} < 0 \end{aligned}$$

Hence, the indirect utility function satisfies single-peakedness on the t dimension for all i, meaning that the tax rate preferred by the median-income voter will be the Condorcet winner tax rate of the fair economy. Replacing ω_i for ω_m in the above expression of $t(\omega_i)$ and knowing that $y_i^* = \omega_i l_i^* = (1-t)\omega_i^2$

for all i, we finally obtain the following expression for the Condorcet winner tax rate:

$$t_m^F = \frac{(1-\lambda)(\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)}{(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}}$$

(b) Just set $\lambda = \delta = 0$ in the expression for t_m^F in order to obtain t_m^S.

Proof of Proposition 2:

The preferred tax rates of the fair and selfish median voters are given by

$$t_m^F = \frac{(1-\lambda)(\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)}{(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}}$$

$$t_m^S = \frac{\overline{y}-y_m}{2\overline{y}-y_m}$$

Taking derivatives,

$$\begin{aligned}\frac{\partial t_m^F}{\partial\lambda} &= \frac{\left[-(\overline{y}-y_m)+\delta(\overline{y}-y_1)\right]\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2}\\ &\quad-\frac{\left[-(2\overline{y}-y_m)+\delta(\overline{y}-y_1)+\overline{y}\right]\left[(1-\lambda)(\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)\right]}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2}\end{aligned}$$

$$\Leftrightarrow \frac{\partial t_m^F}{\partial\lambda} = \frac{\delta(\overline{y}-y_1)\overline{y}-\overline{y}(\overline{y}-y_m)}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2} > 0 \text{ if and only if } \delta > \frac{\overline{y}-y_m}{\overline{y}-y_1}$$

$$\begin{aligned}\frac{\partial t_m^F}{\partial\delta} &= \frac{\left[\lambda(\overline{y}-y_1)\right]\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2}\\ &\quad-\frac{\left[\lambda(\overline{y}-y_1)\right]\left[(1-\lambda)(\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)\right]}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2}\end{aligned}$$

$$\Leftrightarrow \frac{\partial t_m^F}{\partial\delta} = -\frac{\lambda(y_1-\overline{y})\overline{y}}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2} > 0$$

$$\begin{aligned}\frac{\partial t_m^F}{\partial y_m} &= \frac{\left[(1-\lambda)\left(\frac{1}{n}-1\right)+\lambda\delta\frac{1}{n}\right]\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2}\\ &\quad-\frac{\left[(1-\lambda)\left(\frac{2}{n}-1\right)+\lambda\delta\frac{1}{n}+\lambda\frac{1}{n}\right]\left[(1-\lambda)(\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)\right]}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2}\end{aligned}$$

$$\Leftrightarrow \frac{\partial t_m^F}{\partial y_m} = \frac{(1-\lambda)\frac{1}{n}(y_m-n\overline{y})+\lambda\delta\frac{1}{n}y_1}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2} < 0 \text{ if and only if } \frac{(1-\lambda)}{\lambda\delta} > \frac{y_1}{n\overline{y}-y_m}$$

$$\begin{aligned}\frac{\partial t_m^F}{\partial\overline{y}} &= \frac{\left[(1-\lambda)+\lambda\delta\right]\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2}\\ &\quad-\frac{\left[2(1-\lambda)+\lambda\delta+\lambda\right]\left[(1-\lambda)(\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)\right]}{\left[(1-\lambda)(2\overline{y}-y_m)+\lambda\delta(\overline{y}-y_1)+\lambda\overline{y}\right]^2}\end{aligned}$$

$$\Leftrightarrow \frac{\partial t_m^F}{\partial \overline{y}} = \frac{(1-\lambda)\, y_m + \lambda\delta y_1}{\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]^2} > 0$$

$$\begin{aligned}\frac{\partial t_m^F}{\partial y_j}|_{j\neq m,1} \quad &= \quad \frac{\left[(1-\lambda)\frac{1}{n} + \lambda\delta\frac{1}{n}\right]\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]}{\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]^2} \\ &\quad - \frac{\left[(1-\lambda)\frac{2}{n} + \lambda\delta\frac{1}{n} + \lambda\frac{1}{n}\right]\left[(1-\lambda)\left(\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right)\right]}{\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]^2}\end{aligned}$$

$$\Leftrightarrow \frac{\partial t_m^F}{\partial y_j}|_{j\neq m,1} = \frac{(1-\lambda)\frac{1}{n}y_m + \lambda\delta\frac{1}{n}y_1}{\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]^2} > 0$$

$$\begin{aligned}\frac{\partial t_m^F}{\partial y_1} \quad &= \quad \frac{\left[(1-\lambda)\frac{1}{n} + \lambda\delta\left(\frac{1}{n} - 1\right)\right]\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]}{\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]^2} \\ &\quad - \frac{\left[(1-\lambda)\frac{2}{n} + \lambda\delta\left(\frac{1}{n} - 1\right) + \lambda\frac{1}{n}\right]\left[(1-\lambda)\left(\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right)\right]}{\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]^2}\end{aligned}$$

$$\Leftrightarrow \frac{\partial t_m^F}{\partial y_1} = \frac{(1-\lambda)\frac{1}{n}y_m + \lambda\delta\frac{1}{n}\left(y_1 - n\overline{y}\right)}{\left[(1-\lambda)\left(2\overline{y} - y_m\right) + \lambda\delta\left(\overline{y} - y_1\right) + \lambda\overline{y}\right]^2} < 0 \text{ if and only if } \frac{\lambda\delta}{(1-\lambda)} > \frac{y_m}{n\overline{y} - y_1}$$

$$\frac{\partial t_m^S}{\partial y_m} = \frac{\left(\frac{1}{n} - 1\right)\left(2\overline{y} - y_m\right) - \left(\frac{2}{n} - 1\right)\left(\overline{y} - y_m\right)}{\left(2\overline{y} - y_m\right)^2}$$

$$\Leftrightarrow \frac{\partial t_m^S}{\partial y_m} = \frac{\frac{1}{n}\left(y_m - n\overline{y}\right)}{\left(2\overline{y} - y_m\right)^2} < 0$$

$$\frac{\partial t_m^S}{\partial \overline{y}} = \frac{\left(2\overline{y} - y_m\right) - 2\left(\overline{y} - y_m\right)}{\left(2\overline{y} - y_m\right)^2}$$

$$\Leftrightarrow \frac{\partial t_m^S}{\partial \overline{y}} = \frac{y_m}{\left(2\overline{y} - y_m\right)^2} > 0$$

$$\frac{\partial t_m^S}{\partial y_j}|_{j\neq m} = \frac{\frac{1}{n}\left(2\overline{y} - y_m\right) - \frac{2}{n}\left(\overline{y} - y_m\right)}{\left(2\overline{y} - y_m\right)^2}$$

$$\Leftrightarrow \frac{\partial t_m^S}{\partial y_j}|_{j\neq m} = \frac{\frac{1}{n}y_m}{\left(2\overline{y} - y_m\right)^2} > 0$$

Proof of Proposition 3:

(a) Let $t < 1$. Let $\mathbf{y}$ be the pre-tax income vector and let $\mathbf{c}$ be the post-tax income vector. We assumed that the following strict inequality holds: $y_m < \frac{1}{n}\sum_{i=1}^{n} y_i = \overline{y}$. Then, we have that $c_i = (1-t)\,y_i + b$ where $b = \frac{t}{n}\sum_{i=1}^{n} y_i$. Hence, we have that $\sum_{i=1}^{n} c_i = \sum_{i=1}^{n} y_i$. Furthermore, we assumed that $0 < \omega_i < \omega_j < 1$ for $i < j$. As $y_i = (1-t)\,\omega_i^2$ and $c_i = (1-t)^2\,\omega_i^2 + t\overline{y}$, we both have $0 < y_i < y_j$ and $0 < c_i < c_j$ for $i < j$. Therefore, $\mathbf{y} \in \mathbf{X}$ and $\mathbf{c} \in \mathbf{X}$ with $\frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n} c_i = \mu$. After some computations we get

$$c_m - y_m = t\,(\overline{y} - y_m) > 0$$

$$\min\{c_1, ..., c_n\} - \min\{y_1, ..., y_n\} = c_1 - y_1 = t\,(\overline{y} - y_1) > 0$$

Hence, $\mathbf{c}$ strongly median-dominates $\mathbf{y}$.

(b) Let $t < 1$. Let $\mathbf{y}$ be the pre-tax income vector and let $\mathbf{c}$ be the post-tax income vector. Then, $Var(\mathbf{c}) = Var\left[(1-t)\,y_i + b\right] = Var\left[(1-t)\,y_i + t\overline{y}\right]$ where $t\overline{y}$ is constant. Hence, $Var(\mathbf{c}) = (1-t)^2\,Var\,(\mathbf{y}) < Var\,(\mathbf{y})$ as $t < 1$.

Proof of Proposition 4:

(a) If $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{X}$ and $x_m > y_m$, the mean-to-median income ratio is strictly higher under $\mathbf{y}$ than under $\mathbf{x}$ and hence $t_x^S < t_y^S$. If $\mathbf{x}$ strictly median-dominates $\mathbf{y}$ but does not strongly median-dominates $\mathbf{y}$, we have the following two possibilities: (a) $x_m > y_m$ and $x_1 = y_1$, in which case $t_x^F < t_y^F$ as the fair Condorcet winner tax rate is also decreasing in own income (we make the assumption that n is big enough so that $\frac{\partial t_m^F}{\partial y_m} < 0$), and (b) $x_m > y_m$ and $x_1 < y_1$, in which case the relation between t_x^F and t_y^F is undeterminate.

(b) If $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{X}$, $x_m \geq y_m$ and $x_1 \geq y_1$, or both, the mean-to-median income ratio is no less under $\mathbf{y}$ than under $\mathbf{x}$. Hence, $t_x^S \leq t_y^S$. Furthermore, as we know from Proposition 2d and h that the fair Condorcet winner tax rate is decreasing in both own income and the poorest voter's income (i.e. $\frac{\partial t_m^F}{\partial y_m} < 0$ and $\frac{\partial t_m^F}{\partial y_1} < 0$), it follows that $t_x^F < t_y^F$.

(c) In the case of a mean-preserving spread, we can have any combination of relations between x_m and y_m and between x_1 and y_1. Hence, the ranking of the Condorcet winner tax rates in both the selfish and fair economies can be anything.

Proof of Proposition 5:

The preferred tax rate of each voting-income-class is given by

$$t_1 = \frac{(1-\lambda_1)(\overline{y}-y_1)+\lambda_1\delta_1(\overline{y}-y_1)}{(1-\lambda_1)(2\overline{y}-y_1)+\lambda_1\delta_1(\overline{y}-y_1)+\lambda_1\overline{y}} > 0$$

$$t_2 = \frac{(1-\lambda_2)(\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)}{(1-\lambda_2)(2\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)+\lambda_2\overline{y}} > 0$$

$$t_3 = \frac{(1-\lambda_3)(\overline{y}-y_3)+\lambda_3\delta_3(\overline{y}-y_1)}{(1-\lambda_3)(2\overline{y}-y_3)+\lambda_3\delta_3(\overline{y}-y_1)+\lambda_3\overline{y}} \text{ if and only if } \lambda_3 > \frac{(\overline{y}-y_3)}{(\overline{y}-y_3)+\delta_3(y_1-\overline{y})}$$

Then, comparing t_2 and t_2,

$$t_1 > t_2$$

$$\Leftrightarrow \frac{(1-\lambda_1)(\overline{y}-y_1)+\lambda_1\delta_1(\overline{y}-y_1)}{(1-\lambda_1)(2\overline{y}-y_1)+\lambda_1\delta_1(\overline{y}-y_1)+\lambda_1\overline{y}} > \frac{(1-\lambda_2)(\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)}{(1-\lambda_2)(2\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)+\lambda_2\overline{y}}$$

$$\begin{aligned} \Leftrightarrow \quad & [(1-\lambda_1)(\overline{y}-y_1)+\lambda_1\delta_1(\overline{y}-y_1)][(1-\lambda_2)(2\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)+\lambda_2\overline{y}] \\ > \quad & [(1-\lambda_2)(\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)][(1-\lambda_1)(2\overline{y}-y_1)+\lambda_1\delta_1(\overline{y}-y_1)+\lambda_1\overline{y}] \end{aligned}$$

$$\Leftrightarrow (\overline{y}-y_1-\lambda_1\overline{y}+\lambda_1 y_1+\lambda_1\delta_1\overline{y}-\lambda_1\delta_1 y_1)(2\overline{y}-y_2-\lambda_2\overline{y}+\lambda_2 y_2+\lambda_2\delta_2\overline{y}-\lambda_2\delta_2 y_1)$$

$$-(\overline{y}-y_2-\lambda_2\overline{y}+\lambda_2 y_2+\lambda_2\delta_2\overline{y}-\lambda_2\delta_2 y_1)(2\overline{y}-y_1-\lambda_1\overline{y}+\lambda_1 y_1+\lambda_1\delta_1\overline{y}-\lambda_1\delta_1 y_1) > 0$$

$$\Leftrightarrow [\overline{y}+\overline{y}(\lambda_1\delta_1-\lambda_1)+y_1(\lambda_1-\lambda_1\delta_1-1)][2\overline{y}+\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2 y_1]$$

$$-[\overline{y}+\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2 y_1][2\overline{y}+\overline{y}(\lambda_1\delta_1-\lambda_1)+y_1(\lambda_1-\lambda_1\delta_1-1)] > 0$$

Note that, apart from the first terms, the first and the fourth brackets, as well as the second and the third brackets, are respectively the same. Hence, we have

$$\Leftrightarrow \overline{y}[2\overline{y}+\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2 y_1]+2\overline{y}[\overline{y}(\lambda_1\delta_1-\lambda_1)+y_1(\lambda_1-\lambda_1\delta_1-1)]$$

$$-\overline{y}[2\overline{y}+\overline{y}(\lambda_1\delta_1-\lambda_1)+y_1(\lambda_1-\lambda_1\delta_1-1)]-2\overline{y}[\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2 y_1] > 0$$

$$\Leftrightarrow 2\overline{y}+\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2 y_1+2\overline{y}(\lambda_1\delta_1-\lambda_1)+2y_1(\lambda_1-\lambda_1\delta_1-1)$$

$$-2\overline{y}-\overline{y}(\lambda_1\delta_1-\lambda_1)-y_1(\lambda_1-\lambda_1\delta_1-1)-2\overline{y}(\lambda_2\delta_2-\lambda_2)-2y_2(\lambda_2-1)+2\lambda_2\delta_2 y_1 > 0$$

$$\Leftrightarrow -\overline{y}(\lambda_2\delta_2-\lambda_2)-y_2(\lambda_2-1)+\lambda_2\delta_2 y_1+\overline{y}(\lambda_1\delta_1-\lambda_1)+y_1(\lambda_1-\lambda_1\delta_1-1) > 0$$

$$\Leftrightarrow \overline{y}(\lambda_1\delta_1-\lambda_1-\lambda_2\delta_2+\lambda_2)+y_1(\lambda_2\delta_2+\lambda_1-\lambda_1\delta_1-1)+y_2(1-\lambda_2) > 0$$

Doing the same computations for the comparison between t_1 and t_3 yields

$$t_1 > t_3$$

$$\Leftrightarrow \overline{y}(\lambda_1\delta_1-\lambda_1-\lambda_3\delta_3+\lambda_3)+y_1(\lambda_3\delta_3+\lambda_1-\lambda_1\delta_1-1)+y_3(1-\lambda_3) > 0$$

Finally, the comparison between t_2 and t_3 yields

$$t_2 > t_3$$

$$\Leftrightarrow \frac{(1-\lambda_2)(\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)}{(1-\lambda_2)(2\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)+\lambda_2\overline{y}} > \frac{(1-\lambda_3)(\overline{y}-y_3)+\lambda_3\delta_3(\overline{y}-y_1)}{(1-\lambda_3)(2\overline{y}-y_3)+\lambda_3\delta_3(\overline{y}-y_1)+\lambda_3\overline{y}}$$

$$\begin{aligned} \Leftrightarrow \quad & [(1-\lambda_2)(\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)][(1-\lambda_3)(2\overline{y}-y_3)+\lambda_3\delta_3(\overline{y}-y_1)+\lambda_3\overline{y}] \\ > \quad & [(1-\lambda_3)(\overline{y}-y_3)+\lambda_3\delta_3(\overline{y}-y_1)][(1-\lambda_2)(2\overline{y}-y_2)+\lambda_2\delta_2(\overline{y}-y_1)+\lambda_2\overline{y}] \end{aligned}$$

$$\Leftrightarrow (\overline{y}-y_2-\lambda_2\overline{y}+\lambda_2y_2+\lambda_2\delta_2\overline{y}-\lambda_2\delta_2y_1)(2\overline{y}-y_3-\lambda_3\overline{y}+\lambda_3y_3+\lambda_3\delta_3\overline{y}-\lambda_3\delta_3y_1)$$

$$-(\overline{y}-y_3-\lambda_3\overline{y}+\lambda_3y_3+\lambda_3\delta_3\overline{y}-\lambda_3\delta_3y_1)(2\overline{y}-y_2-\lambda_2\overline{y}+\lambda_2y_2+\lambda_2\delta_2\overline{y}-\lambda_2\delta_2y_1) > 0$$

$$\Leftrightarrow [\overline{y}+\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2y_1][2\overline{y}+\overline{y}(\lambda_3\delta_3-\lambda_3)+y_3(\lambda_3-1)-\lambda_3\delta_3y_1]$$

$$-[\overline{y}+\overline{y}(\lambda_3\delta_3-\lambda_3)+y_3(\lambda_3-1)-\lambda_3\delta_3y_1][2\overline{y}+\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2y_1] > 0$$

Note that, apart from the first terms, the first and the fourth brackets, as well as the second and the third brackets, are respectively the same. Hence, we have

$$\Leftrightarrow \overline{y}[2\overline{y}+\overline{y}(\lambda_3\delta_3-\lambda_3)+y_3(\lambda_3-1)-\lambda_3\delta_3y_1]+2\overline{y}[\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2y_1]$$

$$-\overline{y}[2\overline{y}+\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2y_1]-2\overline{y}[\overline{y}(\lambda_3\delta_3-\lambda_3)+y_3(\lambda_3-1)-\lambda_3\delta_3y_1] > 0$$

$$\Leftrightarrow 2\overline{y}+\overline{y}(\lambda_3\delta_3-\lambda_3)+y_3(\lambda_3-1)-\lambda_3\delta_3y_1+2\overline{y}(\lambda_2\delta_2-\lambda_2)+2y_2(\lambda_2-1)-2\lambda_2\delta_2y_1$$

$$-2\overline{y}-\overline{y}(\lambda_2\delta_2-\lambda_2)-y_2(\lambda_2-1)+\lambda_2\delta_2y_1-2\overline{y}(\lambda_3\delta_3-\lambda_3)-2y_3(\lambda_3-1)+2\lambda_3\delta_3y_1 > 0$$

$$\Leftrightarrow -\overline{y}(\lambda_3\delta_3-\lambda_3)-y_3(\lambda_3-1)+\lambda_3\delta_3y_1+\overline{y}(\lambda_2\delta_2-\lambda_2)+y_2(\lambda_2-1)-\lambda_2\delta_2y_1 > 0$$

$$\Leftrightarrow \overline{y}(\lambda_2\delta_2-\lambda_2-\lambda_3\delta_3+\lambda_3)+y_1(\lambda_3\delta_3-\lambda_2\delta_2)+y_2(\lambda_2-1)+y_3(1-\lambda_3) > 0$$

To summarize, we will have

(i)

$$t_1 > t_2 \text{ if and only if}$$

$$\overline{y}(\lambda_1\delta_1-\lambda_1-\lambda_2\delta_2+\lambda_2)+y_1(\lambda_2\delta_2+\lambda_1-\lambda_1\delta_1-1)+y_2(1-\lambda_2) > 0$$

(ii)

$$t_1 > t_3 \text{ if and only if}$$

$$\overline{y}(\lambda_1\delta_1-\lambda_1-\lambda_3\delta_3+\lambda_3)+y_1(\lambda_3\delta_3+\lambda_1-\lambda_1\delta_1-1)+y_3(1-\lambda_3) > 0$$

(iii)

$$t_2 > t_3 \text{ if and only if}$$

$$\overline{y}(\lambda_2\delta_2-\lambda_2-\lambda_3\delta_3+\lambda_3)+y_1(\lambda_3\delta_3-\lambda_2\delta_2)+y_2(\lambda_2-1)+y_3(1-\lambda_3) > 0$$

Proof of Proposition 7:

In order to prove Proposition 7, we have to check the different cases in order to see whether the median voter is still the median skill voter. If not, the redistributive outcome is altered in comparison with the case of an homogeneous population regarding political preferences.

1. In the case SSF, we know that $t_1^S > t_2^S$ and that $t_1^S > t_1^F > t_2^F > t_3^F$. Hence, we might have either $t_1^S > t_2^S > t_3^F$ or $t_1^S > t_3^F > t_2^S$ in which case the fair-rich voter wins. Hence we need

$$t_2^S < t_3^F$$

$$\Leftrightarrow \lambda > \frac{(y_2 - y_3)}{(\overline{y} - y_3) + \delta (y_1 - \overline{y})}$$

2. In the case FSF, we know that $t_2^F > t_2^S$ if and only if $\delta > \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$. Hence we might have either $t_1^F > t_2^S > t_3^F$ or $t_1^F > t_3^F > t_2^S$. In the first case, the median voter and median skill individuals coincide. In the second case, voter 3 becomes the median voter, and we have the same condition as in SSF for this to happen (i.e. $t_3^F > t_2^S$):

$$\lambda > \frac{(y_2 - y_3)}{(\overline{y} - y_3) + \delta (y_1 - \overline{y})}$$

If we have instead that $\delta < \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$, then it means that $t_2^F < t_2^S$. In this case, we might have $t_2^S > t_1^F > t_3^F$, which means that the poor fair voter becomes the median voter. Hence we need

$$t_2^S > t_1^F$$

$$\Leftrightarrow \lambda (1 - \delta) > \frac{y_2 - y_1}{\overline{y} - y_1}$$

3. In the case FFS, we know that $t_1^F > t_2^F > t_3^F \geq t_3^S$. Hence, voter 2 remains the median voter.

4. In the case FSS, we know that $t_1^F > t_2^F > t_2^S > t_3^S$ if and only if $\delta > \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$. In this case, voter 2 remains the median voter.

If we have instead that $\delta < \frac{y_2 - \overline{y}}{y_1 - \overline{y}}$, then it means that $t_2^F < t_2^S$. In this case, we might have $t_2^S > t_1^F > t_3^S$, which means that the poor fair voter becomes the median voter. Hence we need, as in case 2,

$$\lambda (1 - \delta) > \frac{y_2 - y_1}{\overline{y} - y_1}$$

5. In the case SFF we know that $t_1^S > t_1^F > t_2^F > t_3^F$. Hence, voter 2 remains the median voter.

6. In the case SFS we know that $t_1^S > t_1^F > t_2^F > t_3^F \geq t_3^S$. In this case too, voter 2 remains the median voter.